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Time-Frequency Analysis of Gelfand- Shilov-Roumieu Spaces

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Abstract

The present paper aims at realizing a thorough time-frequency analysis of some spaces of rapidly decreasing functions and of ultradistribution spaces. We extend the inversion formula for short time Fourier transform to the general spaces of GSR ultradistributions.

Key words: *Gelfand –Shilov-Roumieu space, inductive limit topological vector space*

Introduction

In our paper we do the time-frequency analysis of some spaces of rapidly decreasing functions and of their duals, which are ultradistribution spaces. Such spaces were firstly introduced and studied by Gelfand and Shilov in [1] and Roumieu in [7]. More precisely, we shall examine the relationship between the growth properties of a GSR function (distribution) at infinity and those of its short time Fourier transform. The obtained results allow us to extend the inversion formula from [3] to the general spaces of GSR ultradistributions.

Similar results were obtained in [3] for spaces of rapidly decreasing functions and spaces of ultradistribution whose behavior at infinity is of Gevrey type. For a comprehensive introduction in the time frequency analysis of functions and distributions, see [2].

The results presented in our paper were used in [6] for the proof of the correcteness of the definition of a new class of modulation spaces.

The paper is organized as follows: for the convenience of the reader we recall in a first section the main definitions and assumptions we use (see also [5] and [6]). The second section of the paper contains the main results and their proofs.

Preliminaries

As in [5], we shall consider sequences of positive real numbers $(M_p)_p$ who satisfy the following assumptions:

(A1) $M_0 = 1,\ M_1 \geq 1$;

(A2) $M_p^2 \leq M_{p-1} M_{p+1},\ (\forall) p \geq 1$;

(A3) there exists a constant $H_1 \geq 1$ so that

$$M_{p+q} \leq H_1^{p+q} M_p M_q,\ (\forall) p,q \geq 0;$$

(A4) there exists a constant $H_2 \geq 1$ so that

$$\sqrt{p} M_{p-1} \leq H_2 M_p,\ (\forall) p \geq 1.$$

We shall denote with M the associated function to such a sequence, $M : (0,\infty) \to [1,\infty)$,

$$M(r) = \sup_{p\geq 0}(p \ln r - \ln M_p),\ (\forall) r > 0.$$

The following properties of the function M, which can be derived from its definition and (A1)-(A4) will be used: $M(r) = 0,\ (\forall) r \leq M_1$, M is increasing and, consequently,

$$M(r+s) \leq M(2r) + M(2s),\ (\forall) r,s > 0$$

(this is a good substitute for the subadditivity;the subadditivity of the associated function is frequently used in papers devoted to the study of ultradistribution spaces) and

$$M(br) + M(cr) \leq M(ar),\ (\forall) b,c,r > 0 \text{ with } a = H_1 \max(b,c)$$

(see Lemma 5 from [4])

We shall also use the following lemma.

Lemma 1. If the sequence $(M_p)_p$ satisfies (A1) – (A3) and M is its associated function, then

$$\int_0^\infty \mathrm{e}^{-pM(\varepsilon r)}\, \mathrm{d}r < \infty,\ (\forall) \varepsilon > 0,\ (\forall) p \geq 1.$$

For a, $b > 0$, $(N_p)_p$ a sequence having the same properties as $(M_p)_p$ and N its associated function, we put

$$\|\varphi\|_{a,b} = \left\|\varphi\, \mathrm{e}^{N(b|x|)}\right\|_\infty + \left\|\hat{\varphi}\, \mathrm{e}^{M(a|\xi|)}\right\|_\infty$$

and

$$\mathcal{S}_{a,b}(M,N) = \{\varphi \in \mathcal{S};\ \|\varphi\|_{a,b} < \infty\}.$$

were $\hat{\varphi} = \mathcal{F}(\varphi)$ denotes the Fourier transform of the function φ. and, as in [5], for simplicity, φ depends on a single variable. The space $\mathcal{S}(M,N)$ is the topological projective limit of the spaces $\mathcal{S}_{a,b}(M,N)$. We shall denote with $\mathcal{S}'(M,N)$ its dual.

Since the sequences $(M_p)_p$ and $(N_p)_p$ are fixed as their associated function, we shall ommit them sometimes from the notations.

Let us recall now the notations of the fundamental operations in time-frequency analysis, the translation by x and the modulation by ω:

$$T_x f(t) = f(t-x),\ (\forall) t \in \boldsymbol{R},$$

$$M_\omega f(t) = \mathrm{e}^{2\pi i \omega t} f(t),\ (\forall) t \in \boldsymbol{R}.$$

The short time Fourier transform with window g of a function f, $V_g f$, is defined by the formula

$$V_g f(x,\omega) = \int f(t) \overline{g(t-x)} \mathrm{e}^{-2\pi i t\omega}\, \mathrm{d}t,\ (\forall)(x,\omega) \in \boldsymbol{R}^2.$$

Lemma 2. The operator $M_\omega T_x : \mathcal{S}_{a,b}(M,N) \to \mathcal{S}_{a/2,b/2}(M,N)$ is a continuous operator, $(\forall)a,b>0$, $(\forall)x,\omega \in \boldsymbol{R}$.

Proof. The proof follows from the estimations

$$\sup_y \left| \mathrm{e}^{N(b|y|/2)} (M_\omega T_x g)(y) \right| \le \sup_y \left| M_\omega T_x (\mathrm{e}^{N(b|x+\cdot|/2)} g)(y) \right| = \sup_y \left| \mathrm{e}^{N(b|x+y|/2)} g(y) \right| \le$$

$$\le \mathrm{e}^{N(b|x|)} \sup_y \left| \mathrm{e}^{N(b|y|)} g(y) \right|$$

and

$$\sup_\eta \left| \mathrm{e}^{M(a|\eta|/2)} (\mathcal{F}(M_\omega T_x g))(\eta) \right| = \sup_\eta \left| \mathrm{e}^{M(a|\eta|/2)} \mathrm{e}^{2\pi i x\omega} (M_{-x} T_\omega \hat{g})(\eta) \right| =$$

$$= \sup_\eta \left| M_{-x} T_\omega (\mathrm{e}^{M(a|\omega+\cdot|/2)} \mathrm{e}^{2\pi i x\omega} \hat{g})(\eta) \right| = \sup_\eta \left| \mathrm{e}^{M(a|\omega+\eta|/2)} \mathrm{e}^{2\pi i x\omega} \hat{g}(\eta) \right| \le$$

$$\le \mathrm{e}^{M(a|\omega|)} \sup_\eta \left| \mathrm{e}^{M(a|\eta|)} \hat{g}(\eta) \right|$$

valid for any function $\varphi \in \mathcal{S}_{a,b}(M,N)$.

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Proposition 1. If $(M_p)_p$, $(N_p)_p$ satisfy (A1)-(A4), then for every $f \in \mathcal{S}'(M,N)$ there exist a, $b>0$ such that for every $g \in \mathcal{S}(M,N)$ there exists a constant $C = C(f,g,a,b)$ so that

$$\left| V_g f(x,\omega) \right| \le C(\mathrm{e}^{M(a|\omega|)} + \mathrm{e}^{N(b|x|)}),\ (\forall)(x,\omega) \in \boldsymbol{R}^2 .$$

Proof. If $g \in \mathcal{S}(M,N)$, then, accordingly to Lemma 2,

$$M_\omega T_x g \in \mathcal{S}_{a/2,b/2}(M,N),\ (\forall)(x,\omega) \in \boldsymbol{R}^2,\ (\forall)a,b>0 .$$

Since $f \in \mathcal{S}'(M,N)$, there exist a, $b>0$ and a constant $C' = C(f)$ such that

$$\left| V_g f(x,\omega) \right| = \left| < f, M_\omega T_x g > \right| \le C' \left[\left\| \mathrm{e}^{N(b/2|\cdot|)} M_\omega T_x g \right\|_\infty + \left\| \mathrm{e}^{M(a/2|\cdot|)} \mathcal{F}(M_\omega T_x g) \right\|_\infty \right] \le$$

$$\le C' \left[\mathrm{e}^{N(b|x|)} \left\| \mathrm{e}^{N(b|\cdot|)} g \right\|_\infty + \mathrm{e}^{M(a|\omega|)} \left\| \mathrm{e}^{M(a|\cdot|)} \hat{g} \right\|_\infty \right].$$

The conclusion of the proposition is true for

$$C = C' \left[\left\| \mathrm{e}^{N(b|\cdot|)} g \right\|_\infty + \left\| \mathrm{e}^{M(a|\cdot|)} \hat{g} \right\|_\infty \right].$$

Proposition 2. If $(M_p)_p$, $(N_p)_p$ satisfy (A1)-(A4), then for every $a_1, a_2, b_1, b_2 > 0$ there exists some constants $a = a(a_1,a_2), b = b(b_1,b_2)$,

$$\lim_{\min(a_1,a_2)\to 0} a(a_1,a_2) = 0,\quad \lim_{\min(b_1,b_2)\to 0} b(b_1,b_2) = 0,$$

$$\lim_{a_1,a_2\to\infty} a(a_1,a_2) = \infty,\quad \lim_{b_1,b_2\to\infty} b(b_1,b_2) = \infty$$

and a positive constant $C = C(a_1, a_2, b_1, b_2)$ such that

$$\left|V_g\varphi(x,\omega)\right| \le C\|\varphi\|_{a_2,b_2}\|g\|_{a_1,b_1}\,\mathrm{e}^{-M(a|\omega|)}\mathrm{e}^{-N(b|x|)},\ (\forall)(x,\omega)\in \boldsymbol{R}^2,$$

$$(\forall)g\in S_{a_1,b_1},\ (\forall)\varphi\in S_{a_2,b_2}.$$

Proof. Let $g\in S_{a_1,b_1}$, $\varphi\in S_{a_2,b_2}$. Then

$$\left|V_g\varphi(x,\omega)\right| = \left|\int_{\boldsymbol{R}}\varphi(t)\overline{g}(t-x)\mathrm{e}^{-2\pi i\omega t}\,\mathrm{d}t\right| \le \int_{\boldsymbol{R}}\left|\varphi(t)\overline{g}(t-x)\right|\mathrm{d}t \le$$

$$\le \left\|\mathrm{e}^{N(b_2|\cdot|)}\varphi\right\|_\infty \left\|\mathrm{e}^{N(b_1|\cdot|)}g\right\|_\infty \int_{\boldsymbol{R}}\mathrm{e}^{-N(b_2|t|)}\mathrm{e}^{-N(b_1|t-x|)}\,\mathrm{d}t =$$

$$= \|\varphi\|_{a_2,b_2}\|g\|_{a_1,b_1}\int_{\boldsymbol{R}}\mathrm{e}^{-N(b_2|t|)}\mathrm{e}^{-N(b_1|t-x|)}\,\mathrm{d}t\ .$$

We have to estimate the last integral. We shall divide it into two integrals:

$$\int_{\boldsymbol{R}}\mathrm{e}^{-N(b_2|t|)}\mathrm{e}^{-N(b_1|t-x|)}\,\mathrm{d}t = \int_{|t-x|\le|x|/2}\mathrm{e}^{-N(b_2|t|)}\mathrm{e}^{-N(b_1|t-x|)}\,\mathrm{d}t + \int_{|t-x|\ge|x|/2}\mathrm{e}^{-N(b_2|t|)}\mathrm{e}^{-N(b_1|t-x|)}\,\mathrm{d}t\ .$$

If $|t-x|\le\dfrac{|x|}{2}$, then $|t|\ge\dfrac{|x|}{2}$ and $N(b_2|t|)\ge N\left(\dfrac{b_2}{2}|t|\right)$. Therefore

$$\int_{\boldsymbol{R}}\mathrm{e}^{-N(b_2|t|)}\mathrm{e}^{-N(b_1|t-x|)}\,\mathrm{d}t \le \int_{|t-x|\le|x|/2}\mathrm{e}^{-N((b_2/2)|x|)}\mathrm{e}^{-N(b_1|t-x|)}\,\mathrm{d}t + \int_{|t-x|\ge|x|/2}\mathrm{e}^{-N(b_2|t|)}\mathrm{e}^{-N((b_1/2)|t-x|)}\,\mathrm{d}t \le$$

$$\le\left(\mathrm{e}^{-N((b_2/2)|x|)}+\mathrm{e}^{-N((b_1/2)|x|)}\right)\int_{\boldsymbol{R}}\left(\mathrm{e}^{-N(b_2|t|)}+\mathrm{e}^{-N(b_1|t|)}\right)\mathrm{d}t\ .$$

Due to Lemma 1, the last integral is finite. We shall denote it with $C_1 = C_1(b_1, b_2)$. Then

$$\left|V_g\varphi(x,\omega)\right| \le 2C_1\|\varphi\|_{a_2,b_2}\|g\|_{a_1,b_1}\,\mathrm{e}^{-N((\min(b_1,b_2)/2)|x|)},\ (\forall)(x,\omega)\in\boldsymbol{R}^2,$$

$$(\forall)g\in S_{a_1,b_1},\ (\forall)\varphi\in S_{a_2,b_2}.$$

If we remark that $V_g\varphi(x,\omega) = V_{\hat{g}}\hat{\varphi}(\omega,-x),\ (\forall)(x,\omega)\in\boldsymbol{R}^2$ (see [2]), we obtain, by performing a similar computation, that

$$\left|V_g\varphi(x,\omega)\right| \le 2C_2\|\varphi\|_{a_2,b_2}\|g\|_{a_1,b_1}\,\mathrm{e}^{-M((\min(a_1,a_2)/2)|\omega|)},\ (\forall)(x,\omega)\in\boldsymbol{R}^2,$$

$$(\forall)g\in S_{a_1,b_1},\ (\forall)\varphi\in S_{a_2,b_2}.$$

for some constant $C_2 = C_2(a_1, a_2)$. Hence

$$\left|V_g\varphi(x,\omega)\right| \le 2\sqrt{C_1C_2}\|\varphi\|_{a_2,b_2}\|g\|_{a_1,b_1}\,\mathrm{e}^{-M((\min(a_1,a_2)/2)|\omega|)}\mathrm{e}^{-N((\min(b_1,b_2)/2)|x|)},$$

$$(\forall)(x,\omega)\in\boldsymbol{R}^2,(\forall)g\in S_{a_1,b_1},\ (\forall)\varphi\in S_{a_2,b_2}.$$

In order to conclude the proof, we observe that since $2M(r) \le M(H_1 r), (\forall) r > 0$, then $M\left(\frac{r}{H_1}\right) \le \frac{1}{2} M(r), (\forall) r > 0$. Therefore we can take

$$a = \frac{1}{2H_1}\min(a_1, a_2) \quad b = \frac{1}{2H_1}\min(b_1, b_2)$$

and $C = 2\sqrt{C_1 C_2}$, where H_1 is a positive constant so that (A3) holds for both sequences, $(M_p)_p$, and $(N_p)_p$.

Proposition 3. If $F : \boldsymbol{R}^2 \to \boldsymbol{C}$ is a continuous function and there exist positive constants $\widetilde{a}, \widetilde{b}$ and $\widetilde{C}$ so that

$$|F(x,\omega)| \le \widetilde{C}(\mathrm{e}^{M(\widetilde{a}|\omega|)} + \mathrm{e}^{N(\widetilde{b}|x|)}),\ (\forall)(x,\omega) \in \boldsymbol{R}^2$$

and if $g \in \mathcal{S}(M,N)$, then

$$f = \iint F(x,\omega) M_\omega T_x g \,\mathrm{d}x\mathrm{d}\omega$$

is in $\mathcal{S}'(M,N)$.

Proof. If $g \in \mathcal{S}_{a_1,b_1}$, $\varphi \in \mathcal{S}_{a_2,b_2}$ and *a*, *b*, *C* are constants as in the conclusion of Proposition 2, then

$$\left|< f, \varphi >\right| \le \iint |F(x,\omega)| \left|< M_\omega T_x g, \varphi >\right| \mathrm{d}x\mathrm{d}\omega \le$$

$$\le C\|\varphi\|_{a_2,b_2}\|g\|_{a_1,b_1} \iint |F(x,\omega)| \mathrm{e}^{-N(b|x|)}\mathrm{e}^{-M(a|\omega|)}\,\mathrm{d}x\mathrm{d}\omega \le$$

$$\le C\widetilde{C}\|\varphi\|_{a_2,b_2}\|g\|_{a_1,b_1} \iint \left(\mathrm{e}^{-N(\widetilde{b}|x|)} + \mathrm{e}^{-M(\widetilde{a}|\omega|)}\right)\mathrm{e}^{-N(b|x|)}\mathrm{e}^{-M(a|\omega|)}\,\mathrm{d}x\mathrm{d}\omega .$$

Using Lemma 1 we can easily deduce that there exist some constants *a* and *b* such that the last sum is finite. Therefore *f* is a continuous functional on $\mathcal{S}(M,N)$.

Proposition 4. If $F : \boldsymbol{R}^2 \to \boldsymbol{C}$ is a continuous function and for every positive constants $\widetilde{a}, \widetilde{b}$, there exists a positive constant $\widetilde{C} = \widetilde{C}(\widetilde{a}, \widetilde{b})$ so that

$$|F(x,\omega)| \le \widetilde{C}(\mathrm{e}^{-M(\widetilde{a}|\omega|)}\mathrm{e}^{-N(\widetilde{b}|x|)}),\ (\forall)(x,\omega) \in \boldsymbol{R}^2$$

and if $g \in \mathcal{S}(M,N)$, then

$$f = \iint F(x,\omega) M_\omega T_x g \,\mathrm{d}x\mathrm{d}\omega$$

is in $\mathcal{S}(M,N)$ and the sum is strongly convergent.

Proof. For every constant $b > 0$ and for every positive constants $\widetilde{a}, \widetilde{b}$, we have

$$\left|\mathrm{e}^{N(b|y|)} \iint F(x,\omega) M_\omega T_x g(y)\,\mathrm{d}x\mathrm{d}\omega\right| \le$$

$$\le \widetilde{C} \iint \mathrm{e}^{-M(\widetilde{a}|\omega|)}\mathrm{e}^{-N(\widetilde{b}|x|)}\mathrm{e}^{N(b|y|)} |M_\omega T_x g(y)|\,\mathrm{d}x\mathrm{d}\omega \le$$

$$\leq \widetilde{C}\sup_z\left(e^{N(2b|z|)}|g(z)|\right)\iint e^{-M(\widetilde{a}|\omega|)}e^{-N(\widetilde{b}|x|)}e^{N(2b|x|)}\,dxd\omega.$$

Using once more Lemma 1, we deduce that for every $b > 0$, there exists a constant $\widetilde{b} > 0$ such that

$$\iint e^{-M(\widetilde{a}|\omega|)}e^{-N(\widetilde{b}|x|)}e^{N(2b|x|)}\,dxd\omega < \infty.$$

Therefore for every $a > 0$, there exists a constant $C = C(a, b) > 0$ such that

$$\left|e^{N(b|y|)}\iint F(x,\omega)M_\omega T_x g(y)\,dxd\omega\right| \leq C\|g\|_{a,2b},\ (\forall)y\in\boldsymbol{R}.$$

In order to estimate the Fourier transform of f, we firstly remark that the integral that defines f is convergent in L^2. Hence, accordingly to Plancherel's theorem, we have that

$$\hat{f}(\xi) = \iint F(x,\omega)\mathcal{F}(M_\omega T_x g)(\xi)\,dxd\omega = \iint F(x,\omega)(M_{-x}T_\omega\hat{g})(\xi)e^{2\pi i x\omega}\,dxd\omega.$$

Arguing as above, we can see that for every a, $b > 0$ there exists another positive constant C such that

$$\left|e^{M(b|\xi|)}\mathcal{F}\left(\iint F(x,\omega)M_\omega T_x g\,dxd\omega\right)(\xi)\right| \leq C\|g\|_{2a,b},\ (\forall)y\in\boldsymbol{R}.$$

The proof is complete.

Theorem 1. A function f which defines a temperate (in Schwartz's sense) distribution is in $\mathcal{S}(M,N)$ if and only if there exists a function $g\in\mathcal{S}(M,N)\setminus\{0\}$ so that for every positive constants $\widetilde{a},\widetilde{b}$, there exists a positive constant $\widetilde{C} = \widetilde{C}(\widetilde{a},\widetilde{b})$ so that

$$|V_g f(x,\omega)| \leq \widetilde{C}(e^{-M(\widetilde{a}|\omega|)}e^{-N(\widetilde{b}|x|)}),\ (\forall)(x,\omega)\in\boldsymbol{R}^2.$$

Proof. If $f, g\in\mathcal{S}(M,N)$ and $\widetilde{a},\widetilde{b} > 0$, then, accordingly to Proposition 2, there exist some constants $a_1, a_2, b_1, b_2 > 0$ so that

$$|V_g\varphi(x,\omega)| \leq C\|\varphi\|_{a_2,b_2}\|g\|_{a_1,b_1}e^{-M(a|\omega|)}e^{-N(b|x|)}),\ (\forall)(x,\omega)\in\boldsymbol{R}^2$$

for some constant $C = C(a_1, a_2, b_1, b_2)$.

On the other hand for a temperate distribution f the inversion formula is valid ([2]):

$$f = \frac{1}{<g,g>}\iint V_g f(x,\omega)M_\omega T_x g\,dxd\omega.$$

Here $<\cdot,\cdot>$ denotes the scalar product in L^2. So, if $V_g f$ satisfies the estimations from above, we can apply Proposition 4 and conclude that $g\in\mathcal{S}(M,N)$.

Remark. One can see from the proof that a function f which defines a temperate distribution is in $\mathcal{S}(M,N)$ if and only if for every function $g\in\mathcal{S}(M,N)$ and every positive constants $\widetilde{a},\widetilde{b}$, there exists a positive constant $\widetilde{C} = \widetilde{C}(\widetilde{a},\widetilde{b};g)$ so that

$$|V_g f(x,\omega)| \leq \widetilde{C}(e^{-M(\widetilde{a}|\omega|)}e^{-N(\widetilde{b}|x|)}),\ (\forall)(x,\omega)\in\boldsymbol{R}^2.$$

Theorem 2 (the inversion formula). If $\gamma, g \in \mathcal{S}(M,N)$ are such that $<\gamma, g> \neq 0$, then

$$f = \frac{1}{<\gamma, g>} \iint V_g f(x,\omega) M_\omega T_x \gamma \,\mathrm{d}x\mathrm{d}\omega,\ (\forall) f \in \mathcal{S}'(M,N).$$

The double integral is weakly convergent.

Proof. Let us temporarily denote with $\tilde{f}$ the distribution defined by the right hand side integral of the formula from above. Then, Proposition 1 and Proposition 3 imply the fact that $\tilde{f} \in \mathcal{S}'(M,N)$ and

$$<\tilde{f}, \varphi> = \frac{1}{<\gamma, g>} \iint V_g f(x,\omega) < M_\omega T_x \gamma, \varphi > \mathrm{d}x\mathrm{d}\omega,\ (\forall)\varphi \in \mathcal{S}(M,N).$$

But

$$\varphi = \frac{1}{<g, \gamma>} \iint V_\gamma \varphi(x,\omega) M_\omega T_x g \ \mathrm{d}x\mathrm{d}\omega$$

and, accordingly to Proposition 2 and Proposition 4, the integral is strongly convergent in $\mathcal{S}(M,N)$. So

$$<f,\varphi> = \left\langle f, \frac{1}{<g,\gamma>} \iint V_\gamma \varphi(x,\omega) M_\omega T_x g \ \mathrm{d}x\mathrm{d}\omega \right\rangle =$$

$$= \frac{1}{<\gamma, g>} \iint \overline{V_\gamma \varphi(x,\omega)} < f, M_\omega T_x g > \mathrm{d}x\mathrm{d}\omega =$$

$$= \frac{1}{<\gamma, g>} \iint V_g f(x,\omega) \overline{< \varphi, M_\omega T_x \gamma >} \ \mathrm{d}x\mathrm{d}\omega =$$

$$= \frac{1}{<\gamma, g>} \iint V_g f(x,\omega) < M_\omega T_x \gamma, \varphi > \mathrm{d}x\mathrm{d}\omega = <\tilde{f}, \varphi>.$$

The proof is complete.

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Analiza timp-frecvență a spațiilor Gelfand-Shilov-Roumieu

Rezumat

Efectuăm o analiză timp-frecvență amănunțită a unor spații de funcții rapid descrescătoare şi a unor spații de ultradistribuții. Extindem formula de inversiune pentru transformarea Fourier în timp scurt la spații generale de ultradistribuții de tip Gelfand-Shilov-Roumieu.