BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LX No. 2/2008	104-108	Seria Matematică - Informatică - Fizică

On Fractional Wavelet Transform

V.I.R. Niculescu[*], Liana Şandru[**]

[*] Formal la NILPRP, Bucureşti
[**] Universitatea Petrol-Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti
e-mail: liana@upg-ploiesti.ro

Abstract

The wavelet theory is a powerful method for processing different physical and chemical signals. The fractional wavelet can be obtained from the a fractional window. In this brief report we present a new wavelet transform which is obtained from Agnesi window. This class of wavelet transform is a flexible one because dependents on two parameters (dilatation or contraction, position). It cover also combinations of discrete and continuous parameter discretisations (hybrid cases).

Key words: *noncausal fractional wavelet, Fourier transform*

Introduction

In processing signals, a part the well known Fourier methods [10], a more powerful instrument is offered by the wavelet theory.

One important direction for numerical and analytical signal evaluation is founded by wavelet growing theory and applications [1,6,7].

Binary collision approximation and the dynamics model for ion-solid computer simulation need new description of interatomic potentials to reduce time function evaluations.

In this brief report we present a new type of wavelet with k rational components, which unlike other ones (Morlet, etc.) have poles.

Method

We define a general normalized Agnesi window function :

$$Agnesik := d_k \Big/ \left(t^{2k} + a^2\right), \tag{1}$$

where $(k = 2,3,4,..)$ and for $k = 2$ the normlization coefficient is $d_k = \frac{2}{3}\sqrt{3}2^{1/4}\frac{a^{7/2}}{\sqrt{\pi}}$.

For $k = 1$ we obtain the famous Agnesi witch [3,4]:

$$Agnesi := a^3 \Big/ \left(t^2 + a^2\right). \tag{2}$$

In figure 1 the Agnesi window function $(k=2)$ is represented for dilatation $a=1.5$ and normlization coefficient d_2 .

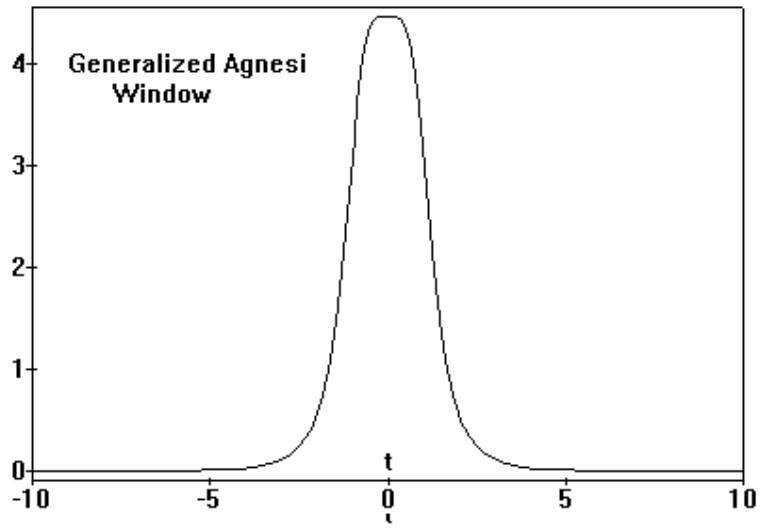

Fig. 1. The Agnesik mother window function

Using this window with poles the speed molecular Maxwellian distribution [9]:

$$\mathrm{d}n = \frac{4}{\sqrt{\pi}} n \left(\frac{m}{2kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) v^2 \,\mathrm{d}v \tag{3}$$

is represented by daughter window :

$$d_k \Big/ \left(\left(t^{2k} - b\right)^2 + a^2\right), \tag{4}$$

where b is the window shift.

For electron speeds in semiconductor lattice [8], we have similar expression :

$$\mathrm{d}W_{v_x} = \left(\frac{m}{2\pi kT}\right)^{1/2} \exp\left(-\frac{mv_x^2}{2kT}\right) \mathrm{d}v_x, \tag{5}$$

where v_x is the x component for identical particles in heat equilibrium.

In paper [11] the ion stopping distribution $f(x, E=0)$ of the projected trajectories is normalized to unity dose. E represent the instant energy for the ion depth between x and $x+\mathrm{d}x$ for the ion incident energy E_0 . For silicium we have:

$$f(x,0) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left(-\frac{\left(x - R_p\right)^2}{2\sigma_p^2}\right), \tag{6}$$

where R_P the ion projected trajectory and σ_p the ion variance. Also we can approximate the exponential in eq. (5) by Agnesi window function which imply less computation time. A similar method is aplicable to generalized ion distributions:

$$f(x,E)=\frac{1}{\sqrt{2\pi}\sigma_p(E)}\exp\left(-\frac{(x-R_p(E))^2}{2\sigma_p^2(E)}\right). \tag{7}$$

In L.S.S. theory we can use the linear approximation:

$$R_p(E)=R_p\{1-E/E_0\},\sigma_p(E)=\sigma_p\{1-E/E_0\}. \tag{8}$$

By construction the mother wavelet is a odd function, which is the product of a gaussian and generalised Agnesi window function $(\text{case}k=2)$:

$$Agnesiw=exp\left(-gt^2\right)t\bullet Agnesik. \tag{9}$$

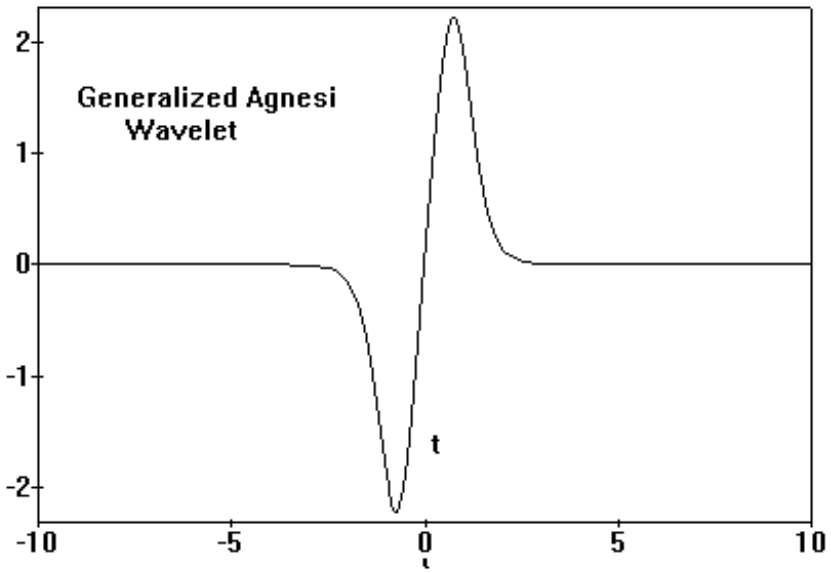

Fig. 2. The Agnesi mother wavelet function $(g=0.5)$

For ion-solid computer simulation there are two approaches: binary collision approximation and the molecular dynamics model [2]. For the energetic particles: the particle energy ranges from the eV to the MeV region. The interaction potentials for the movement of atoms in solids are used for ion bombardment of solids.

The interaction of two atoms is discribed by a potentail function of nuclear charges and the internuclear distance. The screened Coulomb potantials are given by:

$$V(r)=\frac{Z_1Z_2e^2}{r}\Phi\left(\frac{r}{a}\right) \text{ and } \Phi\left(\frac{r}{a}\right)=\sum_{i=1}^{n}c_i\exp\left(-d_i\frac{r}{a}\right),\ \sum_{i=1}^{n}c_i=1. \tag{10}$$

The screening length $a=\left(\frac{9\pi^2}{128}\right)^{1/3}a_B\qquad Z_{12}^{-1/3}$ and $a_B=0.8853$ is the Bohr radius, where $Z=\left(Z_1^x+Z_1^x\right)^y$ and $x=1/2,y=2$. In the range $r\in(0,g)$ using eqs. (9,10) the tail of $\Phi\left(\frac{r}{a}\right)$ for the screened Coulomb potentials was modified:

$$m\Phi\left(\frac{r}{a}\right) = Heaviside(r-g)\,f2m(r),\ f2m(r) = 8\frac{r\text{^}2}{(r\text{^}2+aa\text{^}2)^3} - 2\frac{1}{(r\text{^}2+aa\text{^}2)^2}. \quad (11)$$

The daughter wavelet can be obtained from noncausal mother wavelet with the transformation $r \to r-b$, where b is the position.

Where aa is a parameter and the wavelet $f2$ is the second derivative of Agnesi window (for He we have $aa = 1.5$) . From ε - the relative error g is determined ($g \approx 2.2$):

$$\left|\varepsilon = \Phi\left(\frac{r}{a}\right) - m\Phi\left(\frac{r}{a}\right)\right| \Big/ m\Phi\left(\frac{r}{a}\right). \quad (12)$$

The $m\Phi\left(\frac{r}{a}\right)$ evaluation need only usual arithmetic operations. In figure 3 the homonuclear He modified screened Coulomb potential is given.

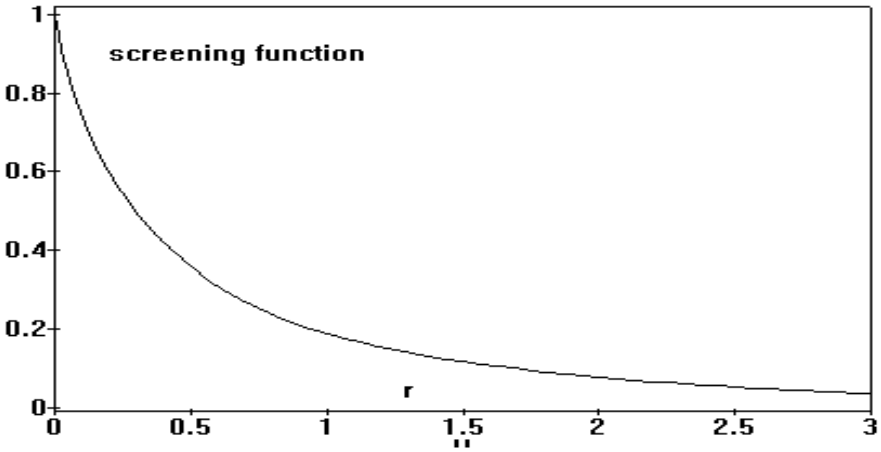

Fig. 3. The Coulomb modified screening function $m\Phi\left(\frac{r}{a}\right)$

The same procedure can be applied to other screening functions and potentials, like Nakagawa and Yamamura [5].

Conclusion

We introduce a new window function with poles, which implies only usual arithmetic operations (addition, substraction, multiplication, division). Also the obtained new fractional daughter wavelet depends of two parameters a ,b (dilatation and shift).

Therefore we constructed a combination of polynom and causal wavelet for screen Coulomb interatomic potentials with less needed computing time for function evaluations.

So a new wavelet transform is a correlation operation between the signal f(t) and the shifted and scaled new mother wavelet. For the same number of parameters this new wavelet evaluation needs less operations than the mexican hat one. So the absence of exponential evaluation for this wavelet reduce the.computing time.

This type of wavelet is more versatile one and can be extended to discrete and continuous parameter discretisations (hybrid cases).

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Transformarea wavelet fracţionară

Rezumat

Teoria wavelet este o metodă puternică de procesare a diferitelor semnale din procesele fizice şi chimice. Waveletul fractionar se poate obţine dintr-o fereastră fracţionară. În acest raport se prezintă o nouă transformare wavelet, care se obţine din fereastra Agnesi. Această clasă de transformări wavelet este una flexibilă, deoarece depinde de doi parametri (dilatare sau contracăie şi poziţie). Sunt avute în vedere şi combinaţiile de parametri discretizaţi şi continui (cazurile hibride).