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The Properties of the Modified Vacuum by a Gravitational Wave

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Abstract

This paper calculates the components of the tensors of the relative gravitational permittivity and of the relative gravitational permittivity of the modified vacuum by a complete gravitational wave. The corresponding tensors of the gravitational permittivity and permeability are non-diagonal and different during the approximation of a second order of the amplitudes powers. There are analyzed the important special cases and the mediated values in space and time.

Key words: *gravitational wave, modified vacuum, gravitational permittivity tensor*

Introduction

Using the developed methods in the following works [2, 3, 6, 7] there can be calculated the components of the tensors of the relative gravitational permittivity and permeability of the modified vacuum by a complete gravitational wave, i.e. a wave which propagates itself along a certain direction and which has composite components in the corresponding metric tensor.

The [2nd and 3rd] works tackled the problem of „the interaction” of two gravitational waves in the approximation of a weak gravitational field. In this approximation, Einstein’s equations of the gravitational field become Maxwell type equations for gravitoelectric ($\vec{E}_g^\mu$, $\vec{D}_g^\mu$) and gravitomagnetic ($\vec{H}_g^\mu$, $\vec{B}_g^\mu$). vectors. In the presence of a gravitational wave, the physical vacuum becomes inhomogeneous and anisotropic. The tensors of the relative gravitational permittivity and permeability are formulated according to the metric tensor $g_{\alpha\beta}$ $(\alpha, \beta = 0, 1, 2, 3)$ and the $g^{\alpha\beta}$ tensor corresponding to the gravitational wave.

In the first part, there we calculate the components of the metric tensor for a gravitational wave which propagates along a certain direction.

In the second part, there we calculate the components of the tensors of the relative gravitational permittivity and permeability.

In the third part, we analyze the problem of the propagation of a gravitational wave in space-time, modified by a gravitational wave and the medium values in time and space of the components of permeability and permittivity tensors.

The Metric Tensor Corresponding to a Gravitational Wave

In order to evaluate the metric tensor which corresponds to a gravitational wave, in the approximation of a weak field, one should write Einstein's equations for formal metrics,

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \tag{1}$$

where $\eta_{\alpha\beta}$ is the metric tensor of Minkowski space and $h_{\alpha\beta}$ are the perturbation of this space determined by the energy tensor of the gravitational wave.

For the (1st) metrics, the Einstein's equations [4, 5]:

$$G^{\alpha\beta} = \left(8\pi G/c^4\right)T^{\alpha\beta}, \tag{2}$$

become

$$\left(\Delta - \partial_t^2\right)\overline{h}^{\alpha\beta} = -\left(16\pi G/c^4\right)T^{\alpha\beta}, \tag{3}$$

with $\partial_t^2 = \partial^2/\partial t^2$,

$$\overline{h}^{\alpha\beta} = h^{\alpha\beta} - (1/2)\eta^{\alpha\beta}h \text{ şi } h \equiv h_\alpha{}^\alpha. \tag{4}$$

The propagation equations of the free waves (without sources, $T^{\alpha\beta} = 0$) have

$$\left(\Delta - \partial_t^2\right)\overline{h}^{\alpha\beta} = 0, \tag{5}$$

the next form

$$\overline{h}^{\alpha\beta} = A^{\alpha\beta}\exp\left(ik_\alpha x^\alpha\right) = A^{\alpha\beta}\exp\left(i\varphi\right) \tag{6}$$

and phase solutions $\varphi = k_\alpha x^\alpha$.

The condition results from the (5) equations with (6) solutions,

$$k_\alpha k^\alpha = \omega_2 - c^2k^2 = \omega^2 - c^2\left(k_x^2 + k_y^2 + k_z^2\right) \tag{7}$$

and so the gravitational waves propagate in the vacuum with the electromagnetic wave velocity $\upsilon_g = c$.

Lorentz' gauge condition

$$\overline{h}^{\alpha\beta}_{,\beta} = 0 \tag{8}$$

(with $\overline{h}^{\alpha\beta}_{,\beta} = \partial\overline{h}^{\alpha\beta}/\partial x_\beta$) implies, for solution type (6), another constraint

$$A^{\alpha\beta}k_\beta = 0. \tag{9}$$

The tensor of dimensionless amplitudes $A^{\alpha\beta}$ is symmetric, so from the 10 different components, the (9) equations allow the definition of 6 independent components. Because the number of the degrees of freedom of gravitational waves, in general relativity is 2, the next constraints must be imposed in order to reduce the number of independent components,

$$A^\alpha{}_\alpha = 0 \tag{10}$$

and

$$A_{\alpha\beta}\, u^\beta = 0. \tag{11}$$

The relations (10) and (11) define the calibration in which the waves are transverse and they have the transverse-traceless gauge if the Lorentz marker is chosen so that the four-vector speed should be

$$u^\beta = \delta_0^\beta. \tag{12}$$

Introducing the (12) expression of the four-vector speed in the relation (11), it results

$$A^{\alpha 0} = 0, \tag{13}$$

i.e. the wave is purely spatial.

The conditions (13) determine the six amplitudes which are different to zero: $A^{ij}, A^{ij} = A^{ji};\ i \neq j;\ i, j = 1, 2, 3$. Choosing A^{11} and A^{22} the two independent amplitudes from the relation (10), with $A_{00} = 0$, it results

$$A^{33} = -\left(A^{11} + A^{22}\right). \tag{14}$$

The three equations which result from (9) determine the three amplitudes $A^{ij} = A^{ji}\ (i \neq j;\ i, j = 1, 2, 3)$ according to the independent amplitudes (A^{11} and A^{22}) and the components of the wave vector $\vec{k}(k_x, k_y, k_z) = \vec{k}(k_1, k_2, k_3)$

$$A^{ij} = -\frac{1}{2k_i k_j}\left[A^{ii}\left(k^2 - k_j^2\right) + A^{jj}\left(k^2 - k_i^2\right)\right]. \tag{15}$$

If the relation (15) is being replaced by the relation (14), it results:

$$A^{12} = -\frac{1}{2k_1k_2}\left[A^{11}\left(k^2 - k_2^2\right) + A^{22}\left(k^2 - k_1^2\right)\right] = -\frac{1}{2k_1k_2}\left[A^{11}k_1^2 + A^{22}k_2^2 + \left(A^{11} + A^{22}\right)k_3^2\right]. \tag{16}$$

$$A^{13} = -\frac{1}{2k_1k_3}\left[A^{11}\left(k^2 - k_3^2\right) + A^{33}\left(k^2 - k_1^2\right)\right] = -\frac{1}{2k_1k_3}\left[A^{11}k_1^2 - \left(A^{11} + A^{22}\right)k_2^2 - A^{22}k_3^2\right]. \tag{17}$$

$$A^{23} = -\frac{1}{2k_2k_3}\left[A^{22}\left(k^2 - k_3^2\right) + A^{33}\left(k^2 - k_2^2\right)\right] = \frac{1}{2k_2k_3}\left[A^{11}k_1^2 - A^{22}k_2^2 - \left(A^{11} + A^{22}\right)k_3^2\right]. \tag{18}$$

In order to determine the corresponding metric tensor (1) to a gravitational wave we should establish the expression of the components of the tensor $h_{\alpha\beta}$ according to the components of the tensor $\bar{h}_{\alpha\beta}$. Therefore we proceed in the following way. From the (4th) expression, in the approximation of first order when $h^{\alpha\beta} = -h_{\alpha\beta}$, it results

$$h_{\alpha\beta} = \bar{h}_{\alpha\beta} - (1/2)\eta_{\alpha\beta}h. \tag{19}$$

The condition (10) and the expression (6) implies

$$\bar{h} = \bar{h}_\alpha{}^\alpha = A_\alpha^\alpha = 0. \tag{20}$$

The definition (4) and the relation (20) determine that the trace of the tensor $h^{\alpha\beta}$ to be zero

$$h = h_\alpha{}^\alpha = 0. \tag{21}$$

Replacing the relation (21) by (19), it results $h_{\alpha\beta} = \bar{h}_{\alpha\beta}$. (22)

The metric tensor expression is obtained by replacing the relation (22) by the (1st) one

$$g_{\alpha\beta} = \eta_{\alpha\beta} + \bar{h}_{\alpha\beta} \tag{23a}$$

or

$$g_{00} = -1,\ g_{0j} = g_{j0} = 0,\ g_{ij} = \delta_{ij} + \bar{h}_{ij} = \delta_{ij} + A_{ij}\exp\left(i\varphi\right). \tag{23b}$$

because the tensors of the gravitational permittivity and permeability may be expressed according to the tensor $g^{\alpha\beta}$ [3], it is necessary to calculate its components.

According to the standard procedure, the components of the tensor $g^{\alpha\beta}$, have the form:

$$g^{00} = -1,\ g^{0j} = g^{j0} = 0, \tag{24a}$$

$$g^{jj} = \frac{\left[1 + A_{nn}\exp\left(i\varphi\right)\right]\left[1 + A_{mm}\exp\left(i\varphi\right)\right] - A_{nm}^2\exp\left(i2\varphi\right)}{\left(-g\right)}, \tag{24b}$$

$$g^{nm}=\frac{\left[1+A_{jj}\exp\left(i\varphi\right)\right]A_{nm}\exp\left(i\varphi\right)-A_{nj}A_{mj}\exp\left(i2\varphi\right)}{\left(-g\right)},\qquad (24c)$$

with $g=\det(g_{\alpha\beta})=-\Big\{\left[1+A_{11}\exp\left(i\varphi\right)\right]\left[1+A_{22}\exp\left(i\varphi\right)\right]\left[1+A_{33}\exp\left(i\varphi\right)\right]+$

$$2A_{12}A_{13}A_{23}\exp\left(i3\varphi\right)-\left[1+A_{11}\exp\left(i\varphi\right)\right]A_{23}^2\exp\left(i2\varphi\right)-$$

$$\left[1+A_{22}\exp\left(i\varphi\right)\right]A_{13}^2\exp\left(i2\varphi\right)-\left[1+A_{33}\exp\left(i\varphi\right)\right]A_{12}^2\exp\left(i2\varphi\right)\Big\},$$

$$\left[1+A_{22}\exp\left(i\varphi\right)\right]A_{13}^2\exp\left(i2\varphi\right)-\left[1+A_{33}\exp\left(i\varphi\right)\right]A_{12}^2\exp\left(i2\varphi\right)\Big\},$$

$j\neq n\neq m=1,2,3$ and Einstein's sum rule is not applied.

The Tensors of the Gravitational Permittivity and Permeability

According to the obtained results in the [3rd, 6th, 7th] works, the expression of the tensor of the relative gravitational permittivity is

$$\varepsilon_{ij}=-\left(-g\right)^{1/2}g^{00}g^{ij}=\left(-g\right)^{1/2}g^{ij},\qquad (25)$$

Replacing in (25) the expressions of the tensor components g^{ij}, given by the relations (24b) and (24c), it results

$$\varepsilon_{jj}=\frac{\left[1+A_{nn}\exp\left(i\varphi\right)\right]\left[1+A_{mm}\exp\left(i\varphi\right)\right]-A_{nm}^2\exp\left(i2\varphi\right)}{\sqrt{-g}},\qquad (26a)$$

$$\varepsilon_{nm}=\frac{\left[1+A_{jj}\exp\left(i\varphi\right)\right]A_{nm}\exp\left(i\varphi\right)-A_{nj}A_{mj}\exp\left(i2\varphi\right)}{\sqrt{-g}},\qquad (26b)$$

with $j\neq n\neq m=1,2,3$ and Einstein's sum rule is not applied.

In the approximation of second order of the tensor components of the amplitudes $A_{\alpha\beta}$, the (26) components have the expressions:

$$\varepsilon_{jj}=1-A_{jj}\exp\left(i\varphi\right)+\frac{1}{2}\left[\left(A_{jj}^2+A_{jn}^2+A_{jm}^2\right)-\left(A_{nn}^2+A_{nn}A_{jj}+A_{nm}^2\right)\right]\exp\left(i2\varphi\right),\qquad (27a)$$

$$\varepsilon_{nm}=-\left[1+A_{jj}\exp\left(i\varphi\right)\right]A_{nm}\exp\left(i\varphi\right)+A_{jn}A_{jm}\exp\left(i2\varphi\right),\qquad (27b)$$

Using the procedure presented in the [3, 6, 7] works, we obtained, through the expressions of the vector the gravitomagnetic intensity

$$\vec{H}_g^{\mu}=\left(-g\right)^{1/2}\left(\vec{u}_1g^{33}g^{22}B_{g1}^{\mu}+\vec{u}_2g^{11}g^{33}B_{g2}^{\mu}+\vec{u}_3g^{22}g^{11}B_{g3}^{\mu}\right)\qquad (28)$$

(with $g=\det\left(g_{\alpha\beta}\right)$) and the tensor relation $B_{gi}^{\mu}=\mu_{ij}H_{gj}^{\mu},\ i,j=1,2,3$, the expression of the tensor of the magnetic gravitational permeability, in the approximation of second power amplitudes of the gravitational wave.

$$\mu_{jj}=\varepsilon_{jj}=\left(-g\right)^{1/2}g^{jj},\qquad (29a)$$

$$\mu_{nm}=\varepsilon_{nm}\left[1+\left(A_{nn}^2+A_{jn}^2+A_{nm}^2\right)\exp\left(i2\varphi\right)\right]\neq\varepsilon_{nm},\quad j\neq m\neq n,\qquad (29b)$$

(Einstein's sum rule is not applied). What results is that the permeability tensor differ from the permeability tensor through the off-diagonal terms.

The two tensors

$$\mu_{jj} = \varepsilon_{jj} = 1 - A_{jj} \exp(i\varphi), \tag{30a}$$

$$\mu_{nm} = \varepsilon_{nm} = -A_{nm} \exp(i\varphi), \tag{30b}$$

with $g = \det(g_{\alpha\beta}) \approx -1$, are identical just in the approximation of first order of amplitudes.

Effects of Metrics Modification Space-Time

The modification space-time determined by the gravitational wave propagation will influence the development of the phenomenon and the existence of other physic systems from this space-time. Next, the influence of the modified metrics over the propagation of another gravitational wave is studied. This phenomenon is considered to be an interaction between two gravitational waves, too.

Given a gravitational wave which propagates on the Oy direction characterized by the gravitoelectromagnetic vectors $\vec{E}_g^{\mu}(0,0,E_{g3}^{\mu})$ and $\vec{H}_g^{\mu}(H_{g1}^{\mu},0,0)$. The two components are functions like $y = x^2$ and t. The differential equation of propagation of the gravitoelectric component E_{g3}^{μ} can be determined from the Maxwell equations type without sources (written in the Gaussian units system):

$$\nabla \vec{D}_g^{\mu} = 0, \tag{31a}$$

$$\nabla \vec{B}_g^{\mu} = 0, \tag{31b}$$

$$\nabla \times \vec{E}_g^{\mu} = -\frac{1}{c}\partial_t \vec{B}_g^{\mu}, \tag{31c}$$

$$\nabla \times \vec{H}_g^{\mu} = \frac{1}{c}\partial_t \vec{D}_g^{\mu}, \tag{31d}$$

with the notation $\partial_t = \partial / \partial t$.

Applying the rotor operator to the equation (31c), it results the equation

$$\nabla \times \left(\nabla \times \vec{E}_g^{\mu}\right) = -\frac{1}{c}\partial_t \left(\nabla \times \vec{B}_g^{\mu}\right). \tag{32a}$$

or

$$\Delta \vec{E}_g^{\mu} = \frac{1}{c}\partial_t \left(\nabla \times \vec{B}_g^{\mu}\right) + \nabla\left(\nabla \vec{E}_g^{\mu}\right). \tag{32b}$$

Because the components E_{g1}^{μ}, E_{g2}^{μ} and the derivative with $z = x^3$ of the component E_{g3}^{μ} ($\partial_3 E_{g3}^{\mu} = \partial E_{g3}^{\mu} / z = 0$) are zero, it results

$$\nabla \vec{E}_g^{\mu} = 0. \tag{33}$$

Replacing (33) by (32b), it results

$$\Delta_3 E_{g3}^{\mu} = \frac{1}{c}\partial_t \left(\nabla \times \vec{B}_g^{\mu}\right)_3 \tag{34}$$

with $\Delta_j = \partial_j^2 = \partial^2 / \partial (x^j)^2, j = 1, 2, 3$ and

$$\left(\nabla \times \vec{B}_g^{\mu}\right)_3 = \partial_1 B_{g2}^{\mu} - \partial_2 B_{g1}^{\mu} = H_{g1}^{\mu}\left(\partial_1 \mu_{12} - \partial_2 \mu_{11}\right) - \mu_{11}\partial_2 H_{g1}^{\mu} \tag{35}$$

because the components H^{μ}_{g2}, H^{μ}_{g3} and the derivative with $x = x^1$ of the component H^{μ}_{g1} ($\partial_1 H^{\mu}_{g1} = 0$) is zero. In order to obtain the searched equation the component H^{μ}_{g1} and its derivative $\partial_2 H^{\mu}_{g1}$ should be eliminated from (35). For this the equation (31d) is used, written per components:

$$\partial_2 H^{\mu}_{g3} - \partial_3 H^{\mu}_{g2} = (1/c)\partial_t D^{\mu}_{g1} \text{ sau } \partial_t\left(\varepsilon_{13} E^{\mu}_{g3}\right) = 0, \tag{36a}$$

$$\partial_3 H^{\mu}_{g1} - \partial_1 H^{\mu}_{g3} = (1/c)\partial_t D^{\mu}_{g2} \text{ sau } \partial_t\left(\varepsilon_{23} E^{\mu}_{g3}\right) = 0, \tag{36b}$$

$$\partial_1 H^{\mu}_{g2} - \partial_2 H^{\mu}_{g1} = (1/c)\partial_t D^{\mu}_{g3} \text{ sau } \partial_2 H^{\mu}_{g1} = -(1/c)\partial_t\left(\varepsilon_{33} E^{\mu}_{g3}\right). \tag{36c}$$

From the equation (32b), in the particular conditions imposed therein before ($H^{\mu}_{g2} = 0$, $H^{\mu}_{g3} = 0$, $\partial_1 H^{\mu}_{g1} = 0, \partial_3 H^{\mu}_{g1} = 0$), it results

$$H^{\mu}_{g1} = -\frac{\mu_{21}}{\left(\partial_1\mu_{11} + \partial_2\mu_{12} + \partial_3\mu_{13}\right)}\partial_2 H^{\mu}_{g1} \ . \tag{37}$$

Replacing the relation (36) by (37) and next the result by (35), it results

$$\left(\nabla \times \vec{B}^{\mu}_{g}\right)_3 = \frac{1}{c}\left[\frac{\left(\partial_1\mu_{12} - \partial_2\mu_{11}\right)}{\left(\partial_1\mu_{11} + \partial_2\mu_{12} + \partial_3\mu_{13}\right)}\mu_{21} + \mu_{11}\right]\partial_t\left(\varepsilon_{33} E^{\mu}_{g3}\right). \tag{38}$$

Replacing the relation (38) by the equation (34), the searched equation is obtained:

$$\begin{aligned}\Delta_3 E^{\mu}_{g3} &= \frac{1}{c^2}\partial_t\left\{\left[\frac{\partial_1\mu_{12} - \partial_2\mu_{11}}{\partial_1\mu_{11} + \partial_2\mu_{12} + \partial_3\mu_{13}}\mu_{21} + \mu_{11}\right]\partial_t\left(\varepsilon_{33} E^{\mu}_{g3}\right)\right\} = \\ &\frac{1}{c^2}\partial_t\left[\left(\mu_{21} M + \mu_{11}\right)\partial_t\left(\varepsilon_{33} E^{\mu}_{g3}\right)\right]\end{aligned} \tag{39a}$$

In order to emphasize the interaction phenomenon between two waves the equation (34) is turned under the next form

$$\begin{aligned}\Delta_3 E^{\mu}_{g3} - \frac{\left(\mu_{11} + M\mu_{21}\right)\varepsilon_{33}}{c^2}\ddot{E}^{\mu}_{g3} &= \frac{2\dot{\varepsilon}_{33}\left(\mu_{11} + M\mu_{21}\right) + \varepsilon_{33}\left(\dot{\mu}_{11} + \mu_{21}\dot{M} + \dot{\mu}_{21} M\right)}{c^2}\dot{E}^{\mu}_{g3} = \\ &\frac{\ddot{\varepsilon}_{33}\left(\mu_{11} + M\mu_{21}\right) + \dot{\varepsilon}_{33}\left(\dot{\mu}_{11} + \mu_{21}\dot{M} + \dot{\mu}_{21} M\right)}{c^2}E^{\mu}_{g3},\end{aligned} \tag{39b}$$

where are noted $\partial_t C = \dot{C}$ and $\partial_t^2 C = \ddot{C}$.

The equation under the form (39b) points out to the propagation speed of the gravitoelectric component on the Oy direction and the corresponding refractive index:

$$\upsilon_{gE2} = \frac{c}{\sqrt{\left(\mu_{11} + M\mu_{21}\right)\varepsilon_{33}}}, \tag{40}$$

$$n_{gE2} = \sqrt{\left(\mu_{11} + M\mu_{21}\right)\varepsilon_{33}} = \sqrt{\varepsilon_{33}\mu_{11}\left(1 + M\mu_{21}/\mu_{11}\right)} = \sqrt{\varepsilon_{33}\mu_{11}}\sqrt{1 + M\mu_{21}/\mu_{11}}\ ; \tag{41}$$

the expression of the gravitational conductivity:

$$\sigma_{gE} = \left(\frac{1}{c^2}\right)\left[2\dot{\varepsilon}_{33}\left(\mu_{11} + M\mu_{21}\right) + \varepsilon_{33}\left(\dot{\mu}_{11} + \mu_{21}\dot{M} + \dot{\mu}_{21} M\right)\right] \square \left(\frac{1}{c^2}\right)\left(2\mu_{11}\dot{\varepsilon}_{33} + \dot{\mu}_{11}\varepsilon_{33}\right) \tag{42}$$

and the expression of the minimum frequency ω_0 analogous to the cut-off frequency in a wave guide:

$$\omega_{0E} = \left(1/c\right)\sqrt{\ddot{\varepsilon}_{33}\left(\mu_{11} + M\mu_{21}\right) + \dot{\varepsilon}_{33}\left(\dot{\mu}_{11} + \mu_{21}\dot{M} + \dot{\mu}_{21}M\right)} \cong \left(1/c\right)\sqrt{\mu_{11}\ddot{\varepsilon}_{33} + \dot{\mu}_{11}\dot{\varepsilon}_{33}} \; . \qquad (43)$$

Through the same procedure we obtain the differential equation of the gravitomagnetic component propagation H^{μ}_{g1}:

$$\Delta_1 H^{\mu}_{g1} - \frac{\left(\varepsilon_{33} + N\varepsilon_{23}\right)\mu_{11}}{c^2}\ddot{H}^{\mu}_{g1} = \frac{2\dot{\mu}_{11}\left(\varepsilon_{33} + N\varepsilon_{23}\right) + \mu_{11}\left(\dot{\varepsilon}_{33} + \varepsilon_{23}\dot{N} + \dot{\varepsilon}_{23}N\right)}{c^2}\dot{H}^{\mu}_{g1} = \frac{\ddot{\mu}_{11}\left(\varepsilon_{33} + N\varepsilon_{23}\right) + \dot{\mu}_{11}\left(\dot{\varepsilon}_{33} + \varepsilon_{23}\dot{N} + \dot{\varepsilon}_{23}N\right)}{c^2}H^{\mu}_{g1}, \qquad (44)$$

with $N = \left(\partial_3\varepsilon_{23} - \partial_2\varepsilon_{33}\right) / \left(\partial_1\varepsilon_{13} + \partial_2\varepsilon_{23} + \partial_3\varepsilon_{33}\right)$.

From the equation (44) we get the expression of the propagation speed of the gravitomagnetic component on the *Oy* direction and the refractive index:

$$\upsilon_{gH2} = \frac{c}{\sqrt{\left(\varepsilon_{33} + N\varepsilon_{23}\right)\mu_{11}}}, \qquad (45)$$

$$n_{gH2} = \sqrt{\left(\varepsilon_{33} + N\varepsilon_{23}\right)\mu_{11}} = \sqrt{\varepsilon_{33}\mu_{11}\left(1 + N\varepsilon_{23}/\varepsilon_{33}\right)} = \sqrt{\varepsilon_{33}\mu_{11}}\sqrt{1 + N\varepsilon_{23}/\varepsilon_{33}} \; ; \qquad (46)$$

the expression of the gravitational conduction:

$$\sigma_{gH2} = \left(1/c^2\right)\left[2\dot{\mu}_{11}\left(\varepsilon_{33} + N\varepsilon_{23}\right) + \mu_{11}\left(\dot{\varepsilon}_{33} + \varepsilon_{23}\dot{N} + \dot{\varepsilon}_{23}N\right)\right] \cong \left(1/c^2\right)\left(2\dot{\mu}_{11}\varepsilon_{33} + \mu_{11}\dot{\varepsilon}_{33}\right) \qquad (47)$$

And the expression of the minimum frequency ω_0 same with the cut-off frequency in a wave guide

$$\omega_{0H2} = \left(1/c\right)\sqrt{\ddot{\mu}_{11}\left(\varepsilon_{33} + N\varepsilon_{23}\right) + \dot{\mu}_{11}\left(\dot{\varepsilon}_{33} + \varepsilon_{23}\dot{N} + \dot{\varepsilon}_{23}N\right)} \cong \left(1/c\right)\sqrt{\ddot{\mu}_{11}\varepsilon_{33} + \dot{\mu}_{11}\dot{\varepsilon}_{33}} \; . \qquad (48)$$

Comparing the parameters set corresponding to the two gravitational quantities (E^{μ}_{g3}, H^{μ}_{g1}), it results that they are different through the *M* and *N* terms which contain derivates with the coordinates of the tensors components of the permittivity and permeability. Only when those terms are neglected the two sets are identical and equal with those obtained in the special case studied in the [2nd, 3rd] works. The expression of the refractive index, in the approximation of first order, is for the wave which propagates through the *Oy* direction, identical with the expression obtained in the [1st] work for a state metrics, if $h_{00} = h_{tt} = 0$.

If a physic system is placed in the space-time metrics (23) and has the dimensions $d_1, d_2, d_3 >> \lambda = 2\pi/k$, $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$ and in this system a phenomenon with the time $\Delta t >> 2\pi/\omega$ takes place, the system and the phenomenon will be influenced by a mediate metrics ($< g_{\alpha\beta} >$) in space and time (mediate in the phase $\varphi = \omega t - \vec{k}\vec{r}$):

$$g_{00} = -1, \; g_{0j} = g_{j0} = 0, \; g_{ij} = \delta_{ij} + \left\langle \overline{h}_{ij} \right\rangle = \delta_{ij} + A_{ij}\left\langle \exp\left(i\varphi\right)\right\rangle = \delta_{ij} \, , \qquad (49)$$

which is identical with the Minkowski metrics and so the presence of the gravitational wave shouldn't influence the system and the phenomena. If it is considered that the presence of a wave modifies the propagation speeds of the interactions, it results that the intermediation has to be realized for the tensor components $g^{\alpha\beta}$ and for the permittivity and permeability:

$$\langle \varepsilon_{jj} \rangle = \langle \mu_{jj} \rangle = 1 - A_{jj} \langle \exp(i\varphi) \rangle + (1/2)\Big[\big(A_{jj}^2 + A_{jn}^2 + A_{jm}^2\big) - \big(A_{nn}^2 + A_{nn}A_{jj} + A_{nm}^2\big)\Big]\langle \exp(i2\varphi) \rangle = 1 + (1/4)\Big[\big(A_{jj}^2 + A_{jn}^2 + A_{jm}^2\big) - \big(A_{nn}^2 + A_{nn}A_{jj} + A_{nm}^2\big)\Big] \neq 0, \tag{50a}$$

$$\langle \varepsilon_{nm} \rangle = -\Big[A_{nm} \langle \exp(i\varphi) \rangle + A_{jj}A_{nm} \langle \exp(i2\varphi) \rangle\Big] + A_{jn}A_{jm} \langle \exp(i2\varphi) \rangle = (1/2)\big(A_{jn}A_{jm} - A_{jj}A_{nm}\big) \neq 0., \tag{50b}$$

$$\langle \mu_{nm} \rangle = \langle \varepsilon_{nm} \rangle + \Big\langle \varepsilon_{nm} \big(A_{nn}^2 + A_{jn}^2 + A_{nm}^2\big) \exp(i2\varphi) \Big\rangle \neq \langle \varepsilon_{nm} \rangle \neq 0, \quad j \neq m \neq n, \tag{50c}$$

Because these mediums are not zero, it results that the other quantities which depend on those like refractive index or conductibility, are not zero.

Conclusions

Using the general form of the gravitational wave, it has been set off that the tensors of the gravitational permittivity and permeability are off-diagonal and different in the approximation of second order of power amplitudes. This difference has as effect the fact that gravitoelectric vectors E_{g3}^{μ} and the gravitomagnetic vectors H_{g1}^{μ}, of another wave which propagates through the Oy direction, have different speeds of propagation. As this vector has the same speed, it results that the assessment of the equality of the two speeds represents additional conditions which connect the quantities which characterize a gravitational wave. The further implications of these constraints will be discussed in a future paper.

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Proprietăţile Vacuumului Modificat de o Undă Gravitaţională

Rezumat

În această lucrare se calculează componentele tensorilor permitivităţii gravitaţionale relative şi permeabilităţii gravitaţionale relative ai vacuumului modificat de o undă gravitaţională completă. Tensorii permitivităţii şi permeabilităţii gravitaţionale corespunzători sunt nediagonali şi sunt diferiţi în aproximaţia de ordinul al doilea al puterilor amplitudinilor. Se analizează cazurile particulare semnificative şi valorile mediate în spaţiu şi timp.