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Corrections to the Problem of Gravitational Waves Interaction

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Abstract

This paper corrects the solutions given to the interaction problem of two gravitational waves in the approximation of a low gravity field. Also there are realized other interpretations about the terms from the differential equation of a gravitational wave which propagates within the modified vacuum in the presence of other wave. So a rest mass of the graviton is set off.

Key words: *gravitational waves interaction, weak field gravity, gravitational refractive index*

Introduction

The interaction problem of two gravitational waves in the approximation of weak field gravity is presented in work [2]. The interaction of the two waves is studied by an indirect phenomenon, intermediate by the modification of the space-time metrics, by the presence of a gravitational wave.

The non-Cartesian (non-Euclidian) character of space-time in the presence of a gravitational wave determines the non-linearity of the propagation differential equations of a gravitational wave which propagates in that space. Einstein's equations written in the approximation of a weak field under the form of Maxwell' equations set off the gravitational permittivity and permeability tensors of the modified vacuum (heterogeneously and non isotropic). From the non-linear equation of a wave propagation in the modified vacuum by the presence o a first wave we obtain: a) the correct expression of a gravitational wave speed and of the gravity refractive index compatible with the results previously obtained [7]; b) the expression of the gravitational conductibility is set off; c) thus we demonstrate the term's coefficient proportional to the gravitoelectric field intensity of the wave, E_{g3}^{μ} as being a cut - off frequency (the space zone with a modified vacuum acts like a wave guide) or like the inertia (mechanical inertia) of the quanta attached to the wave. A relation of dispersion $\omega(k)$ for gravitational waves corresponds to this frequency.

The Inference of the Expression of Permittivity and Permeability Tensors

In order to infer the tensor of the gravitational permittivity and the tensor of the gravitational permeability the paper proposes the methods of the works [2, 6 and 7].

Giving the correspondent metrics of a monochromatic without mixed terms of plane gravitational wave, which propagates along a Ox direction,

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \left[1 - A\sin\left(\omega t - kx\right)\right]\mathrm{d}y^2 + \left[1 + A\sin\left(\omega t - kx\right)\right]\mathrm{d}z^2 - c^2\,\mathrm{d}t^2 , \quad (1)$$

with A dimensionless amplitude of the wave and ω its frequency.

These co-variant components $g_{\alpha\beta};\alpha,\beta = 0,1,2,3$ and the contra-variant $g^{\alpha\beta}$ of the associate metric tensor correspond to this metrics:

$$g_{00} = -1,\ g_{11} = 1,\ g_{22} = 1 - A\sin\left(\omega t - kx\right),\ g_{33} = 1 + A\sin\left(\omega t - kx\right), \quad (2a)$$

$$g^{00} = -1,\ g^{11} = 1,\ g^{22} = \left[1 - A\sin\left(\omega t - kx\right)\right]^{-1},\ g^{33} = \left[1 + A\sin\left(\omega t - kx\right)\right]^{-1} . \quad (2b)$$

Accordingly, the methods described through the works [2, 6, 7], Einstein's equations are written in the linear approximation under the form of Maxwell's equation for a weak field gravity. In the presence of a gravity field generated by a metric gravitational wave (1) the gravitoelectric induction vector $\vec{D}_g^{\mu}$, is:

$$\vec{D}_g^{\mu} = \left(-\det g_{\alpha\beta}\right)^{\frac{1}{2}}\left(\vec{u}_1 g^{\alpha 1} + \vec{u}_2 g^{\alpha 2} + \vec{u}_3 g^{\alpha 3}\right) g^{\beta 0} F^{\mu}_{g\,\alpha\beta} \quad (3)$$

where $\vec{u}_i,\ i = 1,2,3$ are the units of coordinate system of a three-dimensional space and $F^{\mu}_{g\,\alpha\beta}$ are the components of a gravitoelectromagnetic field tensor.

The intensity vector of the gravitoelectric field $\vec{E}_g^{\mu}$ has the expression:

$$\vec{E}_g^{\mu} = E^{\mu}_{g\,1}\vec{u}_1 + E^{\mu}_{g\,2}\vec{u}_2 + E^{\mu}_{g\,3}\vec{u}_3 = F^{\mu}_{g\,01}\vec{u}_1 + F^{\mu}_{g\,02}\vec{u}_2 + F^{\mu}_{g\,03}\vec{u}_3 . \quad (4)$$

The tensorial relation of the two vectors is (we use the Gaussian unit system in which the permittivity and the permeability correspond to the relative permittivity and permeability of the SI system [4]):

$$D^{\mu}_{g\,i} = \varepsilon_{ij} E^{\mu}_{g\,j},\ i,j = 1,2,3 . \quad (5)$$

From the (3) (4) and (5) relations, it results

$$\varepsilon_{ij} = \left(-\det g_{\alpha\beta}\right)^{\frac{1}{2}} g^{ij}, \quad \varepsilon_{ij} = \varepsilon_{ji} = 0, i \neq j,, \quad (6a)$$

or

$$\varepsilon_{11} = \left[1 - A^2\sin^2\left(\omega t - kx\right)\right]^{\frac{1}{2}}, \quad (6b)$$

$$\varepsilon_{22} = \left[\frac{1 + A\sin\left(\omega t - kx\right)}{1 - A\sin\left(\omega t - kx\right)}\right]^{\frac{1}{2}}, \quad (6c)$$

$$\varepsilon_{33} = \left[\frac{1 - A\sin(\omega t - kx)}{1 + A\sin(\omega t - kx)} \right]^{\frac{1}{2}}, , \tag{6d}$$

if the metric tensor is diagonal.

According to the results from [6, 7] the intensity of the gravitoelectric field, $\vec{H}_g^{\mu}$, is given by the relation

$$\vec{H}_g^{\mu} = (-g)^{\frac{1}{2}} \left(\vec{u}_1 g^{33} g^{22} F_{g\,32}^{\mu} + \vec{u}_2 g^{11} g^{33} F_{g\,13}^{\mu} + \vec{u}_3 g^{22} g^{11} F_{g\,21}^{\mu} \right), \tag{7a}$$

with $g = \det(g_{\alpha\beta})$.

Because $F_{g\,32}^{\mu} = B_{g\,1}^{\mu}$, $F_{g\,13}^{\mu} = B_{g\,2}^{\mu}$ and $F_{g\,21}^{\mu} = B_{g\,3}^{\mu}$ are the components of the gravitomagnetic intensity vector, $\vec{B}_g^{\mu}$, the relation (7a) can be also written under the form of:

$$\vec{H}_g^{\mu} = (-g)^{\frac{1}{2}} \left(\vec{u}_1 g^{33} g^{22} B_{g\,1}^{\mu} + \vec{u}_2 g^{11} g^{33} B_{g\,2}^{\mu} + \vec{u}_3 g^{22} g^{11} B_{g\,3}^{\mu} \right) \tag{7b}$$

or

$$B_{g\,1}^{\mu} = (-g)^{-\frac{1}{2}} \left(g^{33} g^{22} \right)^{-1} H_{g\,1}^{\mu} = \left[1 - A^2 \sin^2(\omega t - kx) \right]^{\frac{1}{2}} H_{g\,1}^{\mu}. \tag{8a}$$

$$B_{g\,2}^{\mu} = (-g)^{-\frac{1}{2}} \left(g^{11} g^{33} \right)^{-1} H_{g\,2}^{\mu} = \left[\frac{1 + A\sin(\omega t - kx)}{1 - A\sin(\omega t - kx)} \right]^{\frac{1}{2}} H_{g\,2}^{\mu}. \tag{8b}$$

$$B_{g\,3}^{\mu} = (-g)^{-\frac{1}{2}} \left(g^{22} g^{11} \right)^{-1} H_{g\,3}^{\mu} = \left[\frac{1 - A\sin(\omega t - kx)}{1 + A\sin(\omega t - kx)} \right]^{\frac{1}{2}} H_{g\,3}^{\mu}. \tag{8c}$$

Relations (8) are matricially written under the form of:

$$B_{g\,i}^{\mu} = \mu_{ij} H_{g\,j}^{\mu},\ i, j = 1, 2, 3, \tag{9}$$

with $\mu_{ij} = \mu_{ji} = 0, \quad i \neq j$ and

$$\mu_{11} = \left[1 - A^2 \sin^2(\omega t - kx) \right]^{\frac{1}{2}}, \tag{10a}$$

$$\mu_{22} = \left[\frac{1 + A\sin(\omega t - kx)}{1 - A\sin(\omega t - kx)} \right]^{\frac{1}{2}}, \tag{10b}$$

$$\mu_{33} = \left[\frac{1 - A\sin(\omega t - kx)}{1 + A\sin(\omega t - kx)} \right]^{\frac{1}{2}}. \tag{10c}$$

The expression (4, 5, 6) and (8, 9, 10) indicate that an anisotropic modified vacuum characterized by the gravitational permittivity tensor (6) and by the gravitational permeability tensor (10) correspond to the anisotropic metrics (1) associated to a gravitational wave along the Ox direction. These tensors respect the condition expressed by the Wilson - Dicke theory [3, 8] of the gravitational interaction, $\varepsilon_{ij} = \mu_{ij}$.

The Wave Propagation Equation in the Modified Vacuum

In order to emphasize the „interaction” of two gravitational waves, the propagation equation of a gravitational wave is deduced using Maxwell’s equations for the gravitoelectromagnetic field (in the Gaussian system) without sources $T^{\alpha 0} = 0, \quad \vec{T}_\mu = 0$:

$$\nabla \vec{D}_g^\mu = 0, \tag{11}$$

$$\nabla \vec{B}_g^\mu = 0, \tag{12}$$

$$\nabla \times \vec{E}_g^\mu = -\frac{1}{c}\partial_t \vec{B}_g^\mu, \tag{13}$$

$$\nabla \times \vec{H}_g^\mu = \frac{1}{c}\partial_t \vec{D}_g^\mu, \tag{14}$$

with the formula $\partial_t = \partial/\partial t$.

Using the common procedure of inferring the differential equation of Maxwell’s equations waves, the rotor operator $(\nabla \times)$ at the equation left (13) is applied. It results the differential equation of propagation of the gravitoelectric intensity vector $\vec{E}_g^\mu$:

$$\nabla \times \left(\nabla \times \vec{E}_g^\mu\right) = -\frac{1}{c}\partial_t \left(\nabla \times \vec{B}_g^\mu\right) \tag{15a}$$

or

$$\Delta \vec{E}_g^\mu - \nabla\left(\nabla \vec{E}_g^\mu\right) = \frac{1}{c}\partial_t \left(\nabla \times \vec{B}_g^\mu\right). \tag{15b}$$

The gradient of the divergence of the gravitoelectric intensity vector $\vec{E}_g^\mu$ can be also written

$$\nabla\left(\nabla \vec{E}_g^\mu\right) = \vec{u}_j \left(\partial_i \partial_j E_{g\,i}^\mu\right) \tag{16}$$

and according to relations (8) the j component of the gravitomagnetic intensity vector is

$$\left(\nabla \times \vec{B}_g^\mu\right)_j = \varepsilon_{jkl}\partial_k \left(\mu_{lm} H_{g\,m}^\mu\right), \tag{17}$$

with ε_{jkl} the Levi-Civita symbols.

Replacing the relations (16) and (17) in (15b), it results:

$$\Delta \vec{E}_g^\mu = \vec{u}_j \left[\partial_i \left(\partial_j E_{g\,i}^\mu\right) + \varepsilon_{jkl}\frac{1}{c}\partial_t \partial_k \left(\mu_{lm} H_{g\,m}^\mu\right)\right], \tag{18}$$

with the notation $\partial_k = \partial/\partial x_k$.

When matrix (6) and (10) are diagonals, from the (11th) equation, it results:

$$\partial_j E_{g\,i}^\mu = -\hat{\varepsilon}_{ij}^{-1}\left(E_{g\,p}^\mu \partial_r \varepsilon_{rp}\right). \tag{19}$$

Replacing the relation (19) in the equation (18), it results

$$\Delta\vec{E}_g^{\mu} = -\vec{u}_j\left[\partial_i\left(\hat{\varepsilon}_{ij}^{-1}\left(E_{g\,p}^{\mu}\partial_r\varepsilon_{rp}\right)\right)\right] + \vec{u}_j\,\frac{1}{c}\Big[\varepsilon_{j21}\partial_t\left(\mu_{11}\partial_2 H_{g\,1}^{\mu}\right) + \varepsilon_{j31}\partial_t\left(\mu_{11}\partial_3 H_{g\,1}^{\mu}\right) + \varepsilon_{j12}\partial_t\left(\mu_{22}\partial_1 H_{g\,2}^{\mu}\right) + \varepsilon_{j32}\partial_t\left(\mu_{22}\partial_3 H_{g\,2}^{\mu}\right) + \varepsilon_{j13}\partial_t\left(\mu_{33}\partial_1 H_{g\,3}^{\mu}\right) + \varepsilon_{j23}\partial_t\left(\mu_{33}\partial_2 H_{g\,3}^{\mu}\right) + \varepsilon_{j12} H_{g\,2}^{\mu}\partial_t\mu_{22} + \varepsilon_{j13} H_{g\,3}^{\mu}\partial_t\mu_{33}\Big]. \tag{20}$$

The components $H_{g\,i}^{\mu}$ are obtained from the $E_{g\,i}^{\mu}$ components, using the equations (14) and (12) written under the form of:

$$\partial_m H_{gn}^{\mu} - \partial_n H_{gm}^{\mu} = \frac{1}{c}\left[\varepsilon_{1nm}\partial_t\left(\varepsilon_{11}E_{g\,1}^{\mu}\right) + \varepsilon_{2nm}\partial_t\left(\varepsilon_{22}E_{g\,2}^{\mu}\right) + \varepsilon_{3nm}\partial_t\left(\varepsilon_{33}E_{g\,3}^{\mu}\right)\right], \tag{21}$$

$$H_{g\,1}^{\mu}\partial_t\mu_{11} = -\left(\mu_{11}\partial_1 H_{g\,1}^{\mu} + \mu_{22}\partial_2 H_{g\,2}^{\mu} + \mu_{33}\partial_3 H_{g\,3}^{\mu}\right). \tag{22}$$

If the wave which propagates in the modified vacuum is along the Oy direction, the components of the gravitomagnetic intensity vector are:

$$H_{g\,1}^{\mu} \neq 0, \quad H_{g\,2}^{\mu} = 0, \quad H_{g\,3}^{\mu} = 0\,. \tag{23}$$

Taking count of (23) in the equation (20), it results

$$\Delta\vec{E}_g^{\mu} = -\vec{u}_j\left[\partial_i\left(\hat{\varepsilon}_{ij}^{-1}\left(E_{g\,p}^{\mu}\partial_r\varepsilon_{rp}\right)\right)\right] + \vec{u}_j\,\frac{1}{c}\left[\varepsilon_{j21}\partial_t\left(\mu_{11}\partial_2 H_{g1}^{\mu}\right) + \varepsilon_{j31}\partial_t\left(\mu_{11}\partial_3 H_{g1}^{\mu}\right)\right]. \tag{24}$$

From the relations (21) on the conditions (23), it results:

$$\partial_2 H_{g1}^{\mu} = -\frac{1}{c}\partial_t\left(\varepsilon_{33}E_{g\,3}^{\mu}\right), \tag{25a}$$

$$\partial_3 H_{g1}^{\mu} = -\frac{1}{c}\partial_t\left(\varepsilon_{22}E_{g\,2}^{\mu}\right). \tag{25b}$$

Replacing expressions (25) in equation (24), it results:

$$\Delta\vec{E}_g^{\mu} = \vec{u}_1\left[-E_{g\,1}^{\mu}\partial_1\left(\frac{\partial_1\varepsilon_{11}}{\varepsilon_{11}}\right) - \frac{\partial_1\varepsilon_{11}}{\varepsilon_{11}}\partial_1 E_{g\,1}^{\mu}\right] + \vec{u}_2\left\{\left(\frac{-\partial_1\varepsilon_{11}}{\varepsilon_{22}}\right)\partial_2 E_{g\,1}^{\mu} + \frac{1}{c^2}\left[\mu_{11}\varepsilon_{22}\ddot{E}_{g\,2}^{\mu} + \left(\dot{\mu}_{11}\varepsilon_{22} + 2\mu_{11}\dot{\varepsilon}_{22}\right)\dot{E}_{g\,2}^{\mu} + \left(\dot{\mu}_{11}\dot{\varepsilon}_{22} + \mu_{11}\ddot{\varepsilon}_{22}\right)E_{g\,2}^{\mu}\right]\right\} + \vec{u}_3\left\{\left(\frac{-\partial_1\varepsilon_{11}}{\varepsilon_{33}}\right)\partial_3 E_{g\,1}^{\mu} + \frac{1}{c^2}\left[\mu_{11}\varepsilon_{33}\ddot{E}_{g\,3}^{\mu} + \left(\dot{\mu}_{11}\varepsilon_{33} + 2\mu_{11}\dot{\varepsilon}_{33}\right)\dot{E}_{g\,3}^{\mu} + \left(\dot{\mu}_{11}\dot{\varepsilon}_{33} + \mu_{11}\ddot{\varepsilon}_{33}\right)E_{g\,3}^{\mu}\right]\right\}. \tag{26}$$

The conditions, for the gravitoelectric vector components of a plane transverse wave which propagates along the Oy, are:

$$E_{g\,3}^{\mu} \neq 0, \quad E_{g\,1}^{\mu} = 0, \quad E_{g\,2}^{\mu} = 0\,. \tag{27}$$

Given these conditions, the equation (26) becomes:

$$\Delta\vec{E}_g^{\mu} - \frac{\mu_{11}\varepsilon_{33}}{c^2}\ddot{E}_{g3}^{\mu} = \frac{1}{c^2}\left(\dot{\mu}_{11}\varepsilon_{33} + 2\mu_{11}\dot{\varepsilon}_{33}\right)\dot{E}_{g3}^{\mu} + \frac{1}{c^2}\left(\dot{\mu}_{11}\dot{\varepsilon}_{33} + \mu_{11}\ddot{\varepsilon}_{33}\right)E_{g3}^{\mu} \tag{28a}$$

or, in the first approximation order after the amplitude powers A,

$$\Delta\vec{E}_g^{\mu} - \frac{\mu_{11}\varepsilon_{33}}{c^2}\ddot{E}_{g3}^{\mu} = -\frac{1}{c^2}\left[\frac{2\omega A\cos(\omega t - kx)}{1 + A\sin(\omega t - kx)}\right]\dot{E}_{g3}^{\mu} + \frac{1}{c^2}\left[\frac{\omega^2 A\sin(\omega t - kx)}{1 + A\sin(\omega t - kx)}\right]E_{g3}^{\mu}. \tag{28b}$$

Comparing the propagation equation of a wave in a heterogeneously and anisotropic medium, the coefficients of the time derivates of the gravitoelectric intensity represent:

1. The expression of the propagation speed of the gravitoelectric intensity and so of the gravity wave:

$$\upsilon_y = \upsilon_2 = \frac{c}{\left(\mu_{11}\varepsilon_{33}\right)^{\frac{1}{2}}} \tag{29}$$

and the refractive index of the wave along the Oy direction:

$$n_y = n_2 = \left(\mu_{11}\varepsilon_{33}\right)^{\frac{1}{2}} = \left[1 - A\sin(\omega t - kx)\right]^{\frac{1}{2}}, \tag{30}$$

according to the results [3, 7, 8]. When the wave is transverse, the index along a direction depends on the permittivity and permeability along perpendicular directions. The result is compatible with the gravitational refractive index formula obtained in the work [1] for the case when the component $h_{tt} = h_{00} = 0$;

2. The expression of the gravitational conductivity:

$$\sigma_g = \frac{1}{c^2}\left(\dot{\mu}_{11}\varepsilon_{33} + 2\mu_{11}\dot{\varepsilon}_{33}\right)\dot{E}_{g3}^{\mu} = -\frac{1}{c^2}\frac{2\omega A\cos(\omega t - kx)}{1 + A\sin(\omega t - kx)}. \tag{31}$$

3. The expression of the minimum frequency ω_0 analogue to the cut - off frequency in wave guide:

$$\omega_0 = \frac{1}{c^2}\left(\dot{\mu}_{11}\dot{\varepsilon}_{33} + \mu_{11}\ddot{\varepsilon}_{33}\right)^{\frac{1}{2}} = \frac{\omega}{c}\left[\frac{A\sin(\omega t - kx)}{1 + A\sin(\omega t - kx)}\right]^{\frac{1}{2}} \tag{32}$$

and of a relation of gravitational waves dispersion which propagates in the modified vacuum:

$$\omega(k) = \left(\omega_0^2 + c^2k^2\right)^{\frac{1}{2}}. \tag{33}$$

This relation of dispersion corresponds to a phase velocity:

$$\upsilon_{\varphi y} = \upsilon_{\varphi 2} = \frac{\omega(k)}{k} = c\left(1 + \frac{\omega_0^2}{\omega^2}\right)^{\frac{1}{2}} = \frac{c}{\left[1 + A\sin(\omega t - kx)\right]^{\frac{1}{2}}} \tag{34}$$

and a group velocity:

$$\upsilon_{g\,y} = \upsilon_{g\,2} = \frac{\partial\omega(k)}{\partial k} = c\left(1+\frac{\omega_0^2}{\omega^2}\right)^{-\frac{1}{2}} = c\left[1 + A\sin\left(\omega t - kx\right)\right]^{\frac{1}{2}} . \qquad (35)$$

The refractive index corresponding to a group velocity is:

$$n_{g\,y} = \frac{1}{\left[1 + A\sin\left(\omega t - kx\right)\right]^{\frac{1}{2}}} \cong \left[1 - A\sin\left(\omega t - kx\right)\right]^{\frac{1}{2}} . \qquad (36)$$

In the quantum theory, to the ω_0 angular frequency corresponds its own inertia of the quanta of gravitational field which propagate in the modified vacuum, i.e. of the graviton,

$$m_0 = \frac{\hbar\omega_0}{c^2} .$$

In the classical interpretation, for the gravitational wave, the modified vacuum acts as a dispersive medium or as a wave guide with a ω_0 cut - off frequency which affects the transport velocity of an energy propagated by the wave. It is interesting that an analogue phenomenon has been set off, and for the electromagnetic waves which propagate in the modified vacuum, in the work [7]. These results are compatible with the relative phenomenon too according to which a system of two photons, which propagates on different directions, which have their own mass different from zero, about the system connected at the mass centre, even though the two photons have no own mass [5].

Conclusions

The analysis realized through this paper about the problem of the interaction between two gravitational waves corrects the results of the work [2] and it confirms the obtained results at the works [6,]. We intend to develop these results in the next papers for a) a gravitational wave to which corresponds a metrics with mixed terms, b) a wave which propagates in a certain direction; c) in order to analyzed the reciprocal effect of the two waves which interacts (and the second wave produces the vacuum modification in which the first wave) given the dependencies of coordinates and time the immediate effect has to be calculated in space and time over the wave propagation in the modified vacuum; e) to find out the properties of the modified vacuum by a coherent and incoherent waves background (stochastic background) and f) to study the effect for the electromagnetic waves.

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Corecții la Problema Interacțiunii Undelor Gravitaționale

Rezumat

Lucrarea corectează soluțiile date la problema interacțiunii a două unde gravitaționale în aproximația câmpului gravitațional slab. De asemenea se fac noi interpretări asupra termenilor din ecuația diferențială a undei gravitaționale care se propagă în vacuumul modificat de prezența unei alte unde. În acest fel se pune în evidență o masă de repaus a gravitonului.