

# Evaluation of the Effective Viscosity for Oil-Gas Mixture

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## Abstract

*In this paper we propose a reformulation of the conventional theory in which the concept of the relative permeability of the porous media is replaced with the more familiar concept of an effective phase viscosity. In this theory the bubbles move with the oil as they evolve. The resulting systems of equations, using the PVT data, can be reduced to a coupled pair of nonlinear for the dispersed gas fraction and pressure difference, obtaining a second order evolution equation for the pressure difference. This system of equations can also be derived from the usual theory of two-phase flow.*

**Key words:** *fluid flow, Darcy's law, effective viscosity, relative permeability*

## Introduction

Oil without dissolved gas is called dead oil. Oil saturated with dissolved gas is called live oil. The dissolved gas can be considered condensed and live oil is a mixture of miscible liquids, dead crude oil and condensed gas. The mixture of miscible liquids is like glycerin and water with the caveat that the oil and dissolved gas do not mix in all proportions; the fraction of dissolved gas at saturation is a function of temperature and pressure.

## Model Description

In this model we avoid all constitutive equations regarding nucleation rates and bubble growth. In our model we have only foamy oil and dispersed gas and the dispersed gas enters only through its volume ratio. The model combines Darcy's law, with a dependent mobility, a mass conservation law for ideal mixtures together with a constitutive equation governing the evolution of departures from equilibrium solubility. For a flow in non-equilibrium gradients and time derivatives are crucial. The time derivatives which are used here have a material derivative

$$\frac{d}{dt} = \alpha \frac{\partial}{\partial t} = u \cdot \nabla, \quad (1)$$

where  $\alpha$  is the porosity. The continuity equation is given by

$$\frac{d\rho}{d\phi} \frac{d\phi}{dt} + \rho(\phi) \operatorname{div} u = 0, \quad (2)$$

where

$$\rho(\phi) = \rho_g \phi + \rho_p (1 - \phi) \approx \rho_p (1 - \phi), \quad (3)$$

because  $\rho_g \ll \rho_p$  where  $\rho_p$  is the density of live oil which slightly depends on the volume ratio of the dissolved gas. Combining (3) and (2) we find that

$$\frac{d \log(1 - \phi)}{dt} + \text{div} u = 0. \quad (4)$$

Equation (4) restricts the theory to dispersions of low mobility relative to the suspending liquid. In any motion  $u(x, t)$  of the composite which is divergence free [ $\text{div} u = 0$ ] the dispersed gas fraction does not change following the motion of the fluid

$$\frac{d\phi}{dt} = \frac{\partial \psi}{\partial t} + u \Delta \psi = 0.$$

Bubbles rising under gravity would lead to divergence free motions as would any motion of the bubbly mixture in which dispersed bubbles do not nucleate, diffuse or compress. Motions with non-zero divergence satisfy  $\int_v \frac{1}{1 - \phi} \frac{d\phi}{dt} dv = \int_S u \cdot n \cdot dS$  the flux out of any closed volume, over which the div does not sum to zero, it must be non zero.

This is the simplified way in which our theory accounts for nucleation and diffusion.

Turning to Darcy's law, we let it increase in the direction of gravity. Then

$$u = -\lambda [\nabla p - \rho g e_x] \approx \lambda [\nabla p - \rho_p g (1 - \phi) e_x], \quad (5)$$

where

$$\lambda(\phi) = \frac{k}{\mu(\phi)} \quad (6)$$

is the mobility,  $\mu(\phi)$  is the viscosity of live oil with dispersed gas of volume ratio  $\phi$  and  $k(\phi)$  is the permeability. The viscosity  $\mu(\phi)$  increases for two reasons: dispersed gas bubbles and the reduction of live oil viscosity due to outgassing of light components.

## Relative Permeability

Equations (2), (5) and (6) are implied by the usual theory for two phase flow through porous media based on a relative permeability under the assumption that the average liquid and gas velocities are equal. The demonstration of this fact leads also to a formula (24) for a ratio of the relative permeability under the same assumption.

The composite volume average velocity is given by

$$u = U_p + U_g, \quad (7)$$

where

$$\begin{aligned} U_p &= (1 - \phi) u_p \\ U_g &= \phi u_g \end{aligned} \quad (8)$$

are the superficial velocities (volume flow rate over total area) and  $U_p$  is the average oil velocity and  $U_g$  is the average gas velocity.

The mixture density  $\rho(\phi)$  is given by (3) and the composite mass average velocity is given by

$$u_m = \frac{[\rho_p(1-\phi)u_p + \rho_g\phi u_g]}{\rho(\phi)}. \quad (9)$$

The equations expressing the conservation of the mass of each of the two phases are

$$\alpha \frac{\partial \rho_p}{\partial t} (1-\phi) + \nabla \rho_p (1-\phi) u_p = 0 \quad (10)$$

$$\alpha \frac{\partial \rho_g}{\partial t} \phi + \nabla \rho_g \phi u_g = 0. \quad (11)$$

When the average gas and liquid velocities are equal and from (7), (8) and (9)

$$u_m = u = u_p = u_g. \quad (12)$$

After adding (10) and (11), using (12) we get equation (2).

We now introduce the relative permeability and thus show that we may replace the concept of relative permeability with an equivalent concept of effective viscosity, which is a more familiar concept in the general theory of mixtures.

Neglecting gravity and capillarity we have

$$(1-\phi)u_p = U_p = \frac{k \cdot k_{rp}}{\eta_p} \nabla p = -\frac{k}{\mu_p(\phi)} \nabla p \quad (13)$$

$$\phi u_g = U_g = \frac{k \cdot k_{rg}}{\eta_g} \nabla p = -\frac{k}{\mu_g(\phi)} \nabla p. \quad (14)$$

Where  $k$  is the permeability which we will identify as in (6),  $k_{rp}$  and  $k_{rg}$  are the relative permeabilities,  $\eta_p$  and  $\eta_g$  are molecular viscosities and

$$\mu_p \stackrel{\text{def}}{=} \frac{\eta_p}{k_{rp}}, \quad \mu_g \stackrel{\text{def}}{=} \frac{\eta_g}{k_{rg}} \quad (15)$$

are effective viscosities.

The introduction of effective viscosities is nothing more than a definition; the however, this definition implies that the differential flow resistance can be associated to fluid phases and not with the porous media. Observing now that in (5), there is the composite velocity, we may write (5) and (6) neglecting gravity as

$$u = \frac{k}{\mu(\phi)} \nabla p \quad (16)$$

and using (7) together with (13) and (14), we get

$$\frac{1}{\mu(\phi)} = \frac{1}{\mu_p(\phi)} + \frac{1}{\mu_g(\phi)}. \quad (17)$$

Equation (17) and (15) relate the effective viscosity of the gas-liquid mixture to relative permeability.

The consequences of our basic assumption that the dispersed gas moves with the liquid ( $u = u_g = u_p$ ) may be extracted from the above equations. We find that

$$\frac{1}{\mu_p(\phi)} = \frac{1-\phi}{\mu(\phi)} \quad (18)$$

and

$$\frac{1}{\mu_g(\phi)} = \frac{\phi}{\mu(\phi)}. \quad (19)$$

The effective gas and liquid viscosities are completely determined by the effective viscosity of the mixture when  $u = u_g = u_p$ . Moreover, the ratio of relative permeability's is

$$\frac{k_{rp}}{k_{rg}} = \frac{\eta_p \mu_g(\phi)}{\eta_g \mu_p(\phi)} = \frac{1-\phi}{\phi} \frac{\eta_p}{\eta_g} \quad (20)$$

is determined by  $\phi$  and the ratio of molecular viscosities.

When  $u_g \neq u_p$  we may introduce

$$\chi = \frac{u_p}{u_g} \quad (21)$$

and then

$$\frac{k_{rp}}{k_{rg}} = \frac{\eta_p \mu_g(\phi)}{\eta_g \mu_p(\phi)} = \frac{(1-\phi)\eta_p \chi}{\phi \eta_g}. \quad (22)$$

## Data Acquisition

To verify equation (17) we used experimental data for effective viscosity of the gas and oil to calculate the effective viscosity of the gas-liquid mixture. To extract useful data from the scanned graphics we realize a software to transform the scanned data into a list of values. We then use the scanned graphical data represented in Figure 1.

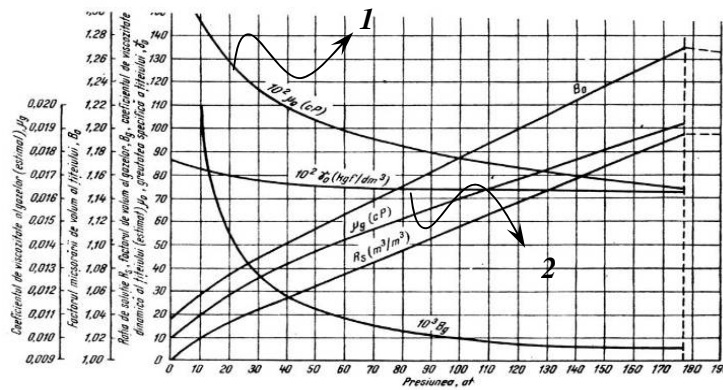


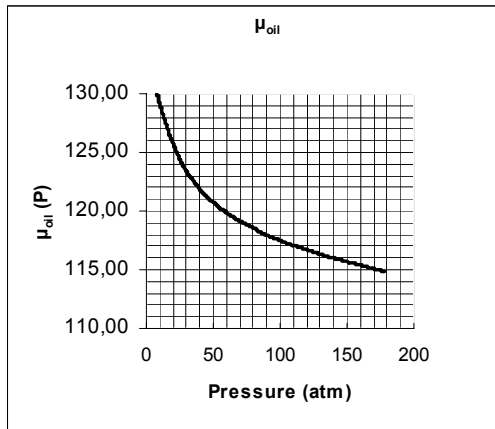
Fig. 1. Experimental data for effective viscosity of the gas and oil

For oil the pressure dependence is represented by curve 1 and for gas by curve 2 (see Figure 1). To realize a useful database it is necessary to scan the image with high resolution and to collect representative data at pixel resolution. This way from the scanned image represented on Figure 1 it is possible to collect up to 1000 values. A part of the collected values are represented in Table 1.

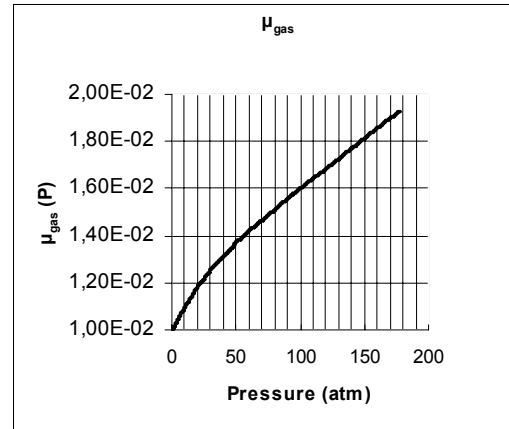
**Table 1.** Extracted data from the scanned graphics

Nr.	Pressure(atm)	$\mu_{oil}$ (P)	$\mu_{gas}$ (P)	$\mu_{mixture}$ (P)	$\Delta\mu=\mu_{gas}-\mu_{mixture}$ (P)
1	7,836290323	129,79271	0,010734317	0,010733429	-8,87693E-07
2	8,006032863	129,7687341	0,010750761	0,01074987	-8,90579E-07
3	8,175775403	129,7406682	0,010767302	0,010766408	-8,93514E-07
...	...	...	...	...	...
...	...	...	...	...	...
1000	177,4090881	114,8989554	0,01926241	0,019259182	-3,22873E-06
1001	177,5788306	114,8972394	0,019264909	0,019261679	-3,22962E-06

Analyzing the graphical representation of the charted values on Figure 2 for oil and Figure 3 for gas, there can be observed the good correlation between scanned and plotted graphics.

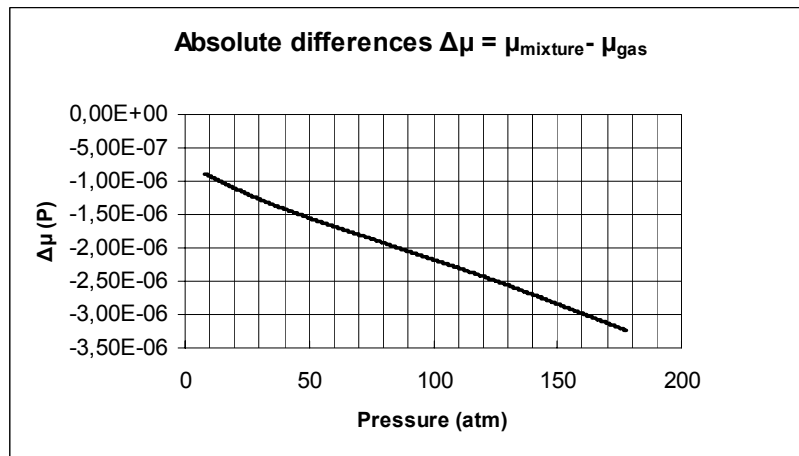


**Fig. 2.** Effective viscosity for oil



**Fig. 3.** Effective viscosity for gas

Because the effective viscosity of gas is appropriate to the effective viscosity of mixture we plotted the differences on Figure 4.



**Fig. 4.** Absolute differences for effective viscosity of oil-gas mixture and gas

## Conclusion

These theoretical results, obtained from equation (17), can be compared to experimental data on effective viscosity on oil-gas mixture. The experimental data are a part of the contract "The Marine Transport of Oil-Gas Mixtures". These comparisons revealed a good correlation between theoretical and experimental data and confirm the validity of this model for these types of oil-gas mixtures, given the limits of experimental errors.

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## Evaluarea vâscozității efective a unui amestec de petrol-gaz

### Rezumat

În această lucrare ne propunem reformularea teoriei convenționale conform căreia în mediile poroase permeabilitatea relativă poate fi înlocuită cu vâscozitatea efectivă a fazei făcând presupunerea că bulele se mișcă în același timp cu petrolul. Utilizăm datele experimentale din diagrama PVT iar sistemul de ecuații rezultat poate fi redus prin cuplarea termenilor neliniari pentru curgerea dispersată a fracției de gaz urmând ca la echilibru să avem o ecuație de gradul doi pentru diferența de presiune. Acest sistem de ecuații poate fi derivat după aceea cu ajutorul teoriei uzuale pentru curgerea trifazică în medii poroase folosind presupunerea amintită.